Math Handbook

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Place Value

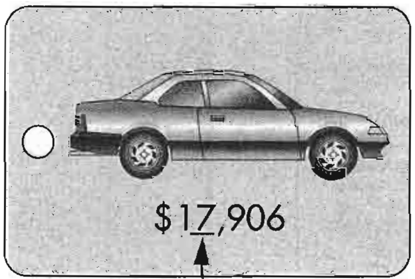
The value of a digit changes depending on its place in a number.

hundred thousands	ten thousands	thousands ,	hundreds	tens	ones

In the two examples below, the digit 7 has different values.

70

The digit 7 in the tens place represents 70.



7,000

The digit 7 in the thousands place represents 7,000.

Look at the values of the digits in this number:

138,405 (one hundred thirty-eight thousand, four hundred five)

the digit 1 represents	100,000
the digit 3 represents	30,000
the digit 8 represents	8,000
the digit 4 represents	400
the digit 0 represents	0 tens
the digit 5 represents	5

138,405 = 100,000 + 30,000 + 8,000 + 400 + 5

Math Words
- **place value**
- **ones**
- **tens**
- **hundreds**
- **thousands**
- **ten thousands**
- **hundred thousands**
- **digit**

What are the values of the digits in the number 106,297?

Place Value of Large Numbers

Math Words

- **million**
- **billion**
- **trillion**
- **googol**

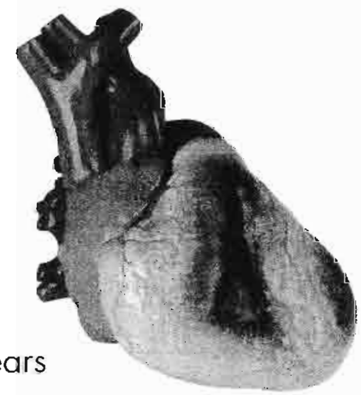
A pattern is used to name very large numbers.

TRILLIONS			BILLIONS			MILLIONS			THOUSANDS			ONES		
___	___	___,	___	___	___,	___	___	___,	___	___	___,	___	___	___
hundred trillions	ten trillions	one trillions	hundred billions	ten billions	one billions	hundred millions	ten millions	one millions	hundred thousands	ten thousands	one thousands	hundreds	tens	ones

Every three digits are separated by a comma.
The three grouped digits share a name (such as "millions").

Within a group of three digits, there is a pattern of ones, tens, and hundreds.

Very large numbers are used to count heartbeats.



(one)	about 1	heartbeat per second
(one thousand)	1,000	heartbeats in less than 20 minutes
(one million)	1,000,000	heartbeats in less than 2 weeks
(one billion)	1,000,000,000	heartbeats in about 35 years

A googol is a very, very large number!
One googol is written as the digit 1 followed by 100 zeros.

10,000,000,000,000,000,000,000,000,000,000,000,000,000,000,
000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,
000,000,000,000,000,000

Addition Strategies (page 1 of 2)

In Grade 5 you are practicing addition strategies.

$$6{,}831 + 1{,}897 =$$

Breaking Apart the Numbers

Rachel solved the problem by adding one number in parts.

Rachel's solution

$$\begin{array}{r} 6{,}831 \\ +1{,}000 \\ \hline 7{,}831 \\ +\ \ 800 \\ \hline 8{,}631 \\ +\ \ \ \ 90 \\ \hline 8{,}721 \\ +\ \ \ \ \ \ 7 \\ \hline \mathbf{8{,}728} \end{array}$$

Charles, Zachary, and Janet solved the problem by adding by place. Their solutions are similar, but they recorded their work differently.

Charles's solution

$$\begin{array}{r} 6{,}831 + 1{,}897 = \phantom{7{,}000} \\ 6{,}000 + 1{,}000 = 7{,}000 \\ 800 + 800 = 1{,}600 \\ 30 + 90 = \phantom{1{,}}120 \\ 1 + 7 = \phantom{1{,}60}8 \\ \hline \mathbf{8{,}728} \end{array}$$

Zachary's solution

$$\begin{array}{r} 6{,}831 \\ +\ 1{,}897 \\ \hline 7{,}000 \\ 1{,}600 \\ 120 \\ 8 \\ \hline \mathbf{8{,}728} \end{array}$$

Janet's solution

$$\begin{array}{r} {\scriptstyle 1\ 1} \\ 6{,}831 \\ +\ 1{,}897 \\ \hline \mathbf{8{,}728} \end{array}$$

Addition Strategies (page 2 of 2)

6,831 + 1,897 =

Changing the Numbers

Cecilia solved the problem by changing one number and adjusting the sum. She changed 1,897 to 2,000 to make the problem easier to solve.

Cecilia's solution

6,831	
+ 2,000	*I added 2,000 instead of 1,897.*
8,831	
- 103	*Then I subtracted the extra 103.*
8,728	

Benito solved the problem by creating an equivalent problem.

Benito's solution

6,831 + 1,897 =
(-3) (+3) *I added 3 to 1,897 and subtracted 3 from 6,831.*

6,828 + 1,900 = **8,728**

Show how you would solve the problem 6,831 + 1,897.

Subtraction Strategies (page 1 of 4)

In Grade 5, you are using different strategies to solve subtraction problems efficiently.

$$\begin{array}{r} 3{,}726 \\ -\ 1{,}584 \\ \hline \end{array}$$

Subtracting in Parts

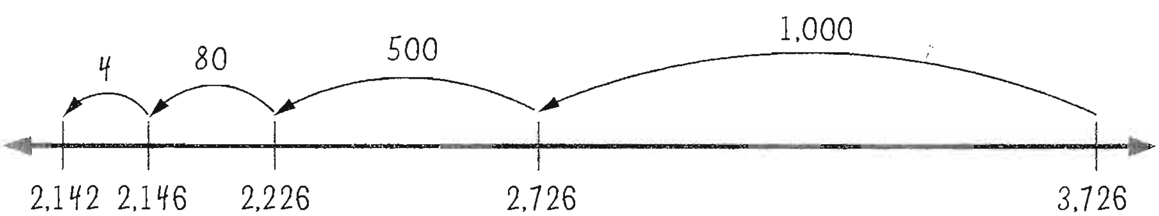
Tamira solved this problem by subtracting 1,584 in parts.

Tamira's solution

$$\begin{array}{r} 3{,}726 \\ -\ 1{,}000 \\ \hline 2{,}726 \\ -\ 500 \\ \hline 2{,}226 \\ -\ 80 \\ \hline 2{,}146 \\ -\ 4 \\ \hline \mathbf{2{,}142} \end{array}$$

I started at 3,726 and jumped back 1,584 in four parts: 1,000, then 500, then 80, and then 4. I landed on 2,142. The answer is the place where I landed.

$3{,}726 - 1{,}584 = \mathbf{2{,}142}$



Subtraction Strategies (page 2 of 4)

$$\begin{array}{r} 3{,}726 \\ -\ 1{,}584 \\ \hline \end{array}$$

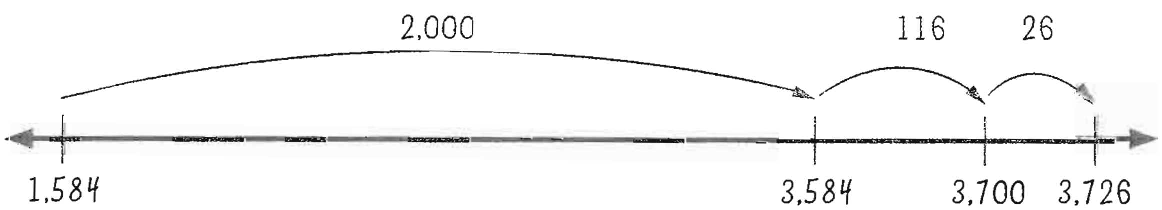
Adding Up

Felix added up from 1,584.

Felix's solution

$$\begin{array}{rcrcl} 1{,}584 & + & \_\_\_\_ & = & 3{,}726 \\ 1{,}584 & + & 2{,}000 & = & 3{,}584 \\ 3{,}584 & + & 116 & = & 3{,}700 \\ 3{,}700 & + & 26 & = & 3{,}726 \\ \hline & & \mathbf{2{,}142} & & \end{array}$$

The answer is the total of all the jumps from 1,584 up to 3,726.

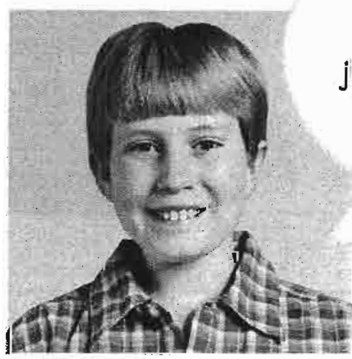


Subtracting Back

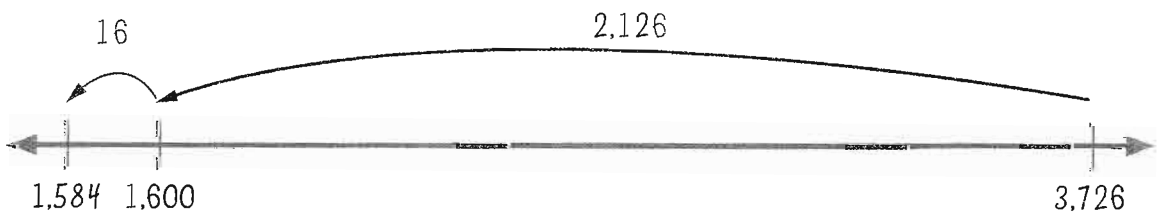
Walter used a subtracting back strategy.

Walter's solution

$$\begin{array}{rcrcl} 3{,}726 & - & 1{,}584 & = & \_\_\_\_ \\ 3{,}726 & - & 2{,}126 & = & 1{,}600 \\ 1{,}600 & - & 16 & = & 1{,}584 \\ \hline & & \mathbf{2{,}142} & & \end{array}$$



The answer is the total of the two jumps from 3,726 back to 1,584.

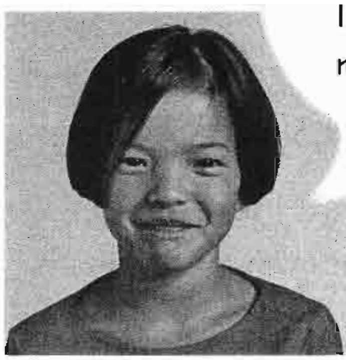


Subtraction Strategies (page 3 of 4)

$$\begin{array}{r} 3{,}726 \\ -\ 1{,}584 \\ \hline \end{array}$$

Changing the Numbers

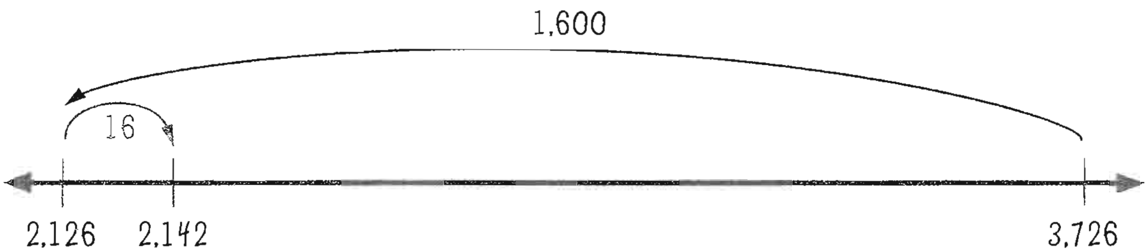
Hana solved the problem by changing one number and adjusting the answer.



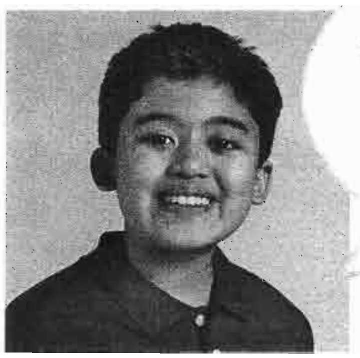
I subtracted 1,600 instead of 1,584. I subtracted too much, so I added 16 back on.

Hana's solution

3,726 − 1,600 = 2,126
2,126 + 16 = **2,142**



Joshua solved the problem by creating an equivalent problem.

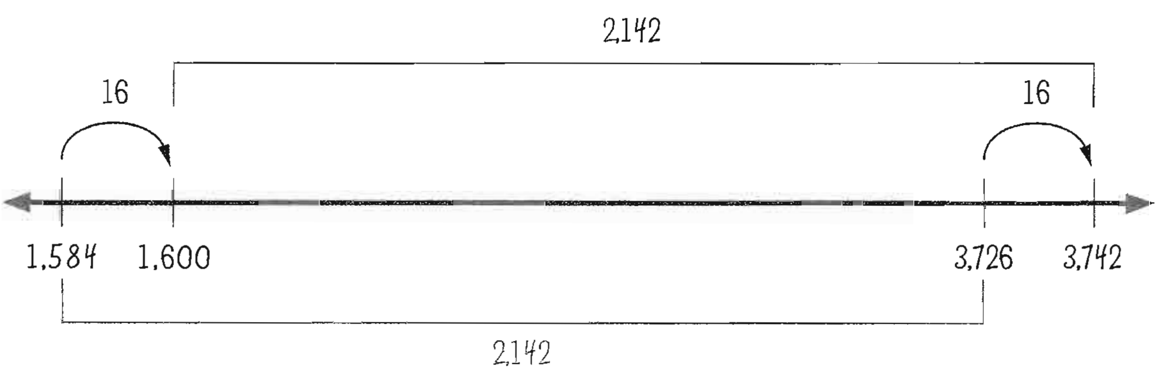


I added 16 to each number. For me, 1,600 is easier to subtract.

Joshua's solution

3,726 − 1,584 =
(+16) (+16)

3,742 − 1,600 = **2,142**



Subtraction Strategies (page 4 of 4)

$$\begin{array}{r} 3{,}726 \\ -\ 1{,}584 \\ \hline \end{array}$$

Subtracting by Place

Yumiko subtracted by place. She combined positive and negative results to find her answer.

Yumiko's solution

$$\begin{array}{r} 3{,}726 \\ -\ 1{,}584 \\ \hline 2 \\ -\ 60 \\ 200 \\ 2{,}000 \\ \hline \mathbf{2{,}142} \end{array}$$

This notation shows each step in Yumiko's solution.

$$\begin{array}{rcrcrcrcl} & 3{,}000 & + & 700 & + & 20 & + & 6 & \\ -\ (& 1{,}000 & + & 500 & + & 80 & + & 4\ ) & \\ \hline & 2{,}000 & + & 200 & + & -60 & + & 2 & = \mathbf{2{,}142} \end{array}$$

Avery subtracted by place, using the U.S. algorithm.

Avery's solution

$$\begin{array}{r} \scriptstyle 6\ \ \ \ \\ 3{,}\not{7}{}^{1}26 \\ -\ 1{,}584 \\ \hline \mathbf{2{,}142} \end{array}$$

This notation shows each step in Avery's solution.

$$\begin{array}{rcrcrcrcl} & & & \scriptstyle 600 & & \scriptstyle 100 & & & \\ & 3{,}000 & + & \not{700} & + & 20 & + & 6 & \\ -\ (& 1{,}000 & + & 500 & + & 80 & + & 4\ ) & \\ \hline & 2{,}000 & + & 100 & + & 40 & + & 2 & = \mathbf{2{,}142} \end{array}$$

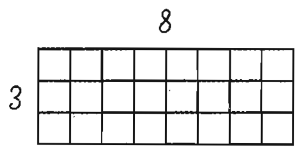
How would you solve the problem 3,726 – 1,584?

Arrays

Math Words
- **array**
- **dimensions**

Arrays can be used to represent multiplication.

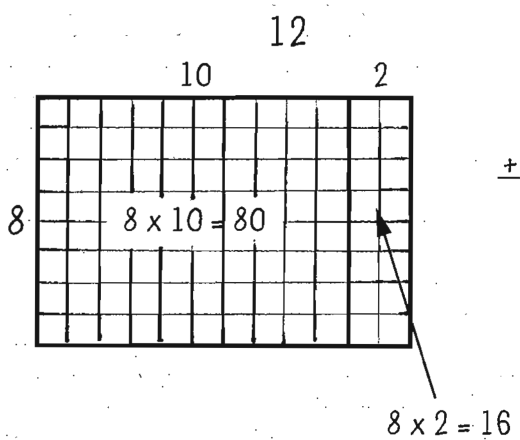
This is one of the rectangular arrays you can make with 24 tiles.



The dimensions of the array are 3 × 8 (or 8 × 3, depending on how you are looking at the array).

This array shows that
- 3 and 8 are two of the factors of 24.
- 24 is a multiple of 8.
- 24 is a multiple of 3.

This array shows a way to solve 8 × 12.



$$\begin{array}{r} 80 \\ +\ 16 \\ \hline \mathbf{96} \end{array}$$

← Use graph paper

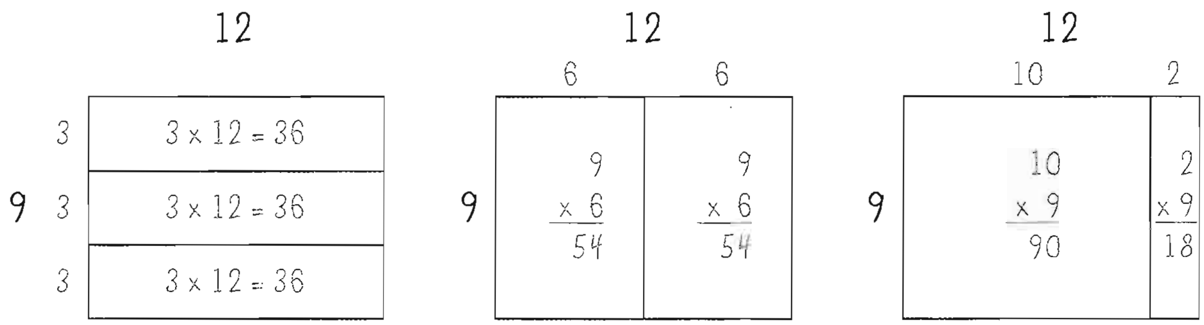
8 × 12 = (8 × 10) + (8 × 2) = 80 + 16 = **96**

Draw an array with dimensions 5 by 9.

Unmarked Arrays

For larger numbers, arrays without grid lines can be easier to use than arrays with grid lines.

Look at how unmarked arrays are used to show different ways to solve the problem 9 × 12.

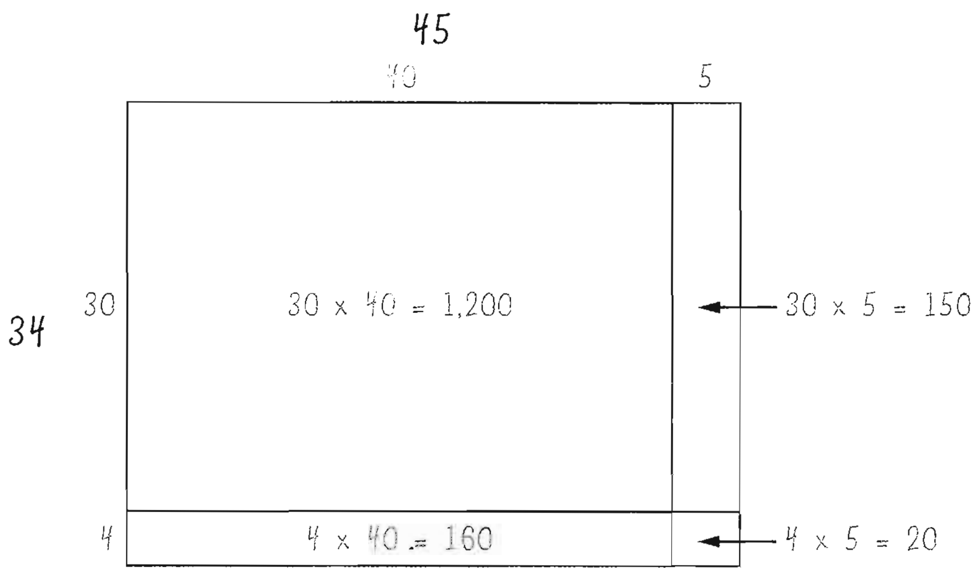


36 + 36 + 36 = **108**

54 + 54 = **108**

90 + 18 = **108**

This unmarked array shows a solution for 34 × 45.



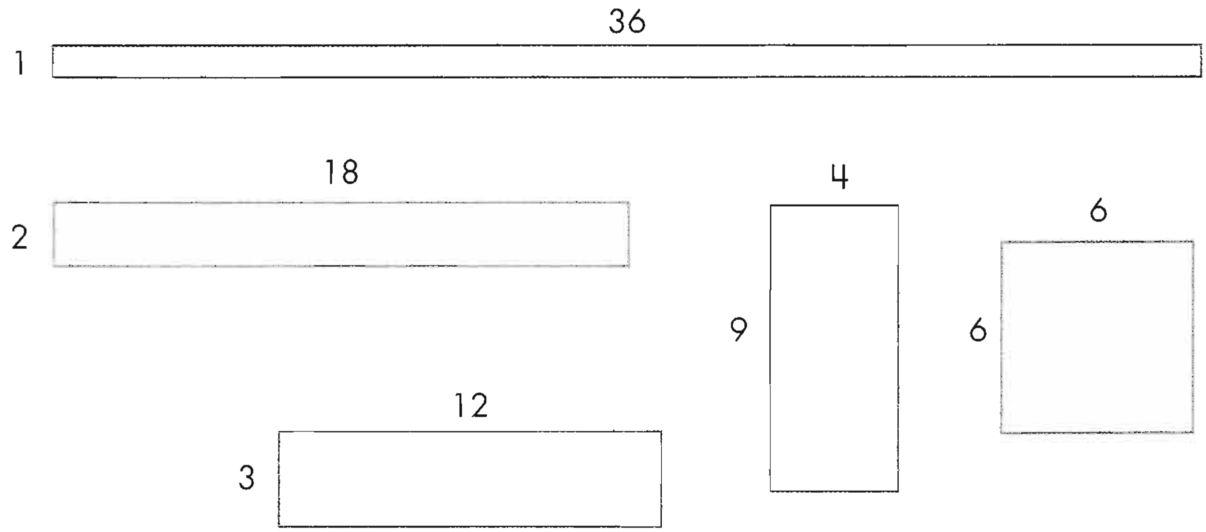
$$\begin{array}{r} 1{,}200 \\ 160 \\ 150 \\ +\ 20 \\ \hline \mathbf{1{,}530} \end{array}$$

34 x 45 = **1,530**

Factors

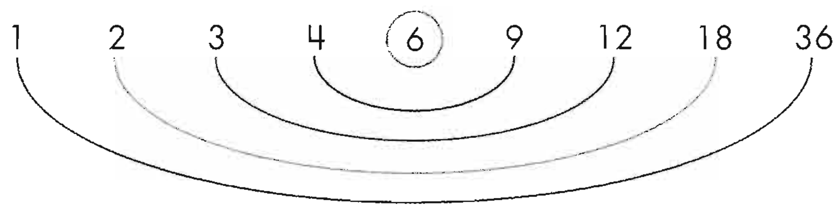
Math Words
- **factor**

These are all the possible whole-number rectangular arrays for the number 36, using whole numbers.



Each dimension of these rectangles is a factor of 36.

Listed in order, the factors of 36 are



Pairs of factors can be multiplied to get a product of 36.

$1 \times 36 = 36$	$2 \times 18 = 36$	$3 \times 12 = 36$
$4 \times 9 = 36$	$6 \times 6 = 36$	$9 \times 4 = 36$
$12 \times 3 = 36$	$18 \times 2 = 36$	$36 \times 1 = 36$

Use the factors of 36 to find the factors of 72.
Use the factors of 36 to find the factors of 360.

Multiples

Math Words
- **multiple**

This 300 chart shows skip counting by 15. The shaded numbers are multiples of 15. A multiple of 15 is a number that can be divided evenly into groups of 15.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200
201	202	203	204	205	206	207	208	209	210
211	212	213	214	215	216	217	218	219	220
221	222	223	224	225	226	227	228	229	230
231	232	233	234	235	236	237	238	239	240
241	242	243	244	245	246	247	248	249	250
251	252	253	254	255	256	257	258	259	260
261	262	263	264	265	266	267	268	269	270
271	272	273	274	275	276	277	278	279	280
281	282	283	284	285	286	287	288	289	290
291	292	293	294	295	296	297	298	299	300

20 × **15** = 300

300 ÷ **15** = 20

Use the shaded numbers on the 300 chart to write other multiplication and division equations about the multiples of 15.

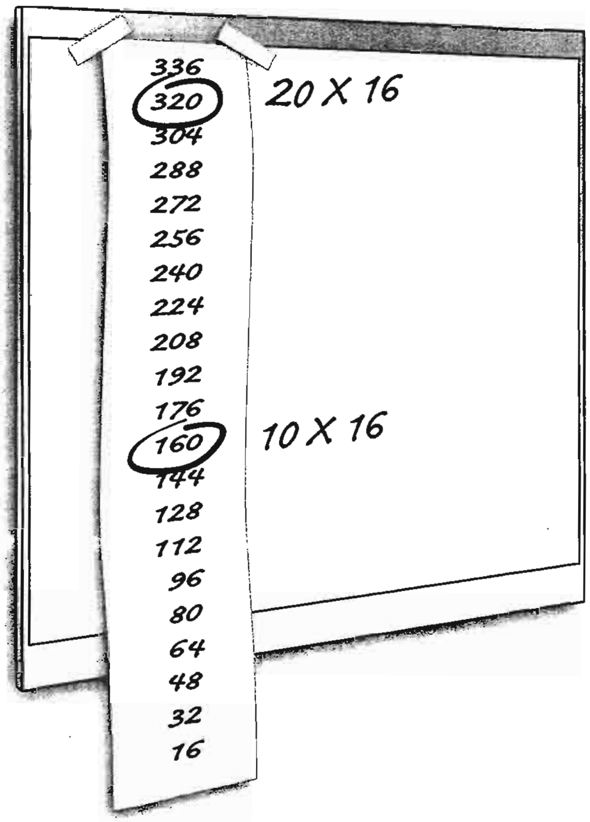
? × 15 = ? ? ÷ 15 = ?

Multiple Towers

When you skip count by a certain number, you are finding multiples of that number.

Nora's class made a multiple tower for the number 16. They recorded the multiples of 16 on a paper strip, starting at the bottom.

They circled every 10th multiple of 16 and used them as landmark multiples to solve the following problems.



21 × 16 = 336

Nora's solution

We know that 20 × 16 = 320.
336 is next on the tower after 320, so it is one more 16.

30 × 16 = 480

Georgia's solution

30 × 16 would be the next landmark multiple on our tower.
Since 3 × 16 = 48,
then 30 × 16 = 48 × 10.

208 ÷ 16 = 13

Renaldo's solution

Ten 16s land on 160.
Three more 16s will go to 208.

How would you use this multiple tower to solve this problem?
18 × 16 = ______

SMH

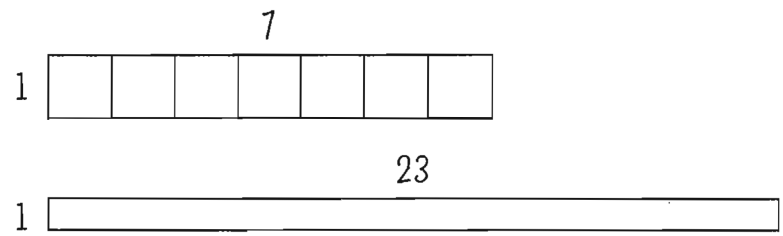
Properties of Numbers

(page 1 of 2)

Math Words
- **prime number**
- **composite number**
- **square number**

When a number is represented as an array, you can recognize some of the special properties of that number.

Prime numbers have exactly two factors: 1 and the number itself.

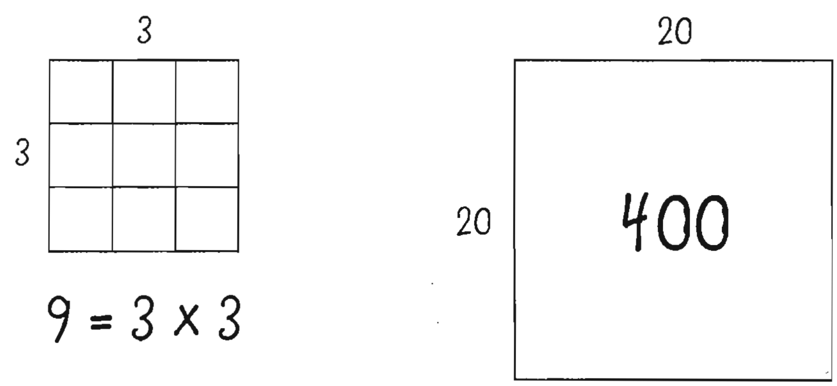


7 and 23 are examples of prime numbers.

Numbers that have more than two factors are called composite numbers.

The number 1 has only one factor. It is neither a prime number nor a composite number.

A square number is the result when a number is multiplied by itself.



$9 = 3 \times 3$

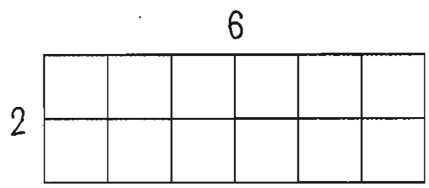
9 and 400 are examples of square numbers.

Properties of Numbers

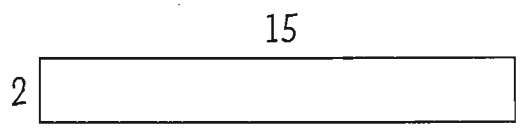
(page 2 of 2)

Math Words
- **even number**
- **odd number**

An even number is composed of groups of 2. One of the factors of an even number is 2.



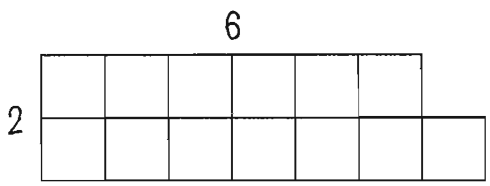
$12 = 2 \times 6$



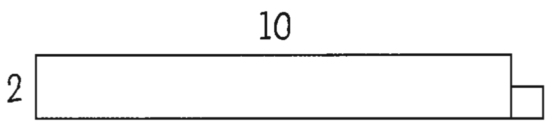
$30 = 2 \times 15$

12 and 30 are examples of even numbers.

An odd number is composed of groups of 2 plus 1. An odd number does not have 2 as a factor.



$13 = (2 \times 6) + 1$



$21 = (2 \times 10) + 1$

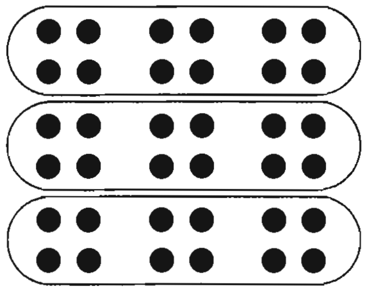
13 and 21 are examples of odd numbers.

Find all the prime numbers up to 50.
Find all the square numbers up to 100.

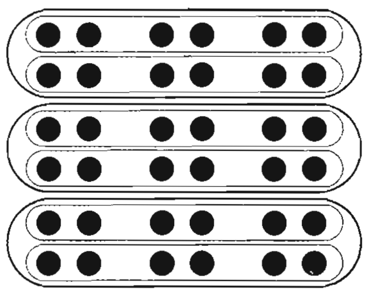
Multiplying More than Two Numbers (page 1 of 2)

There are 36 dots in this arrangement.

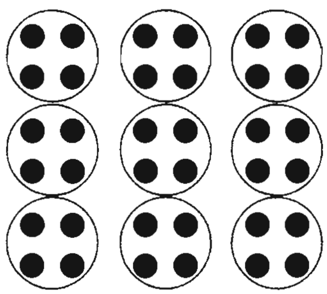
You can visualize the total number of dots in many ways.



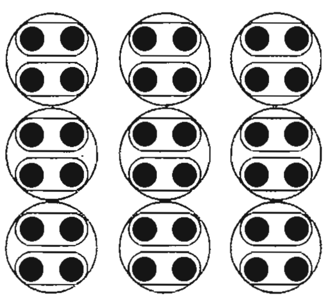
3×12
3 groups of 12



$3 \times (2 \times 6)$
3 groups of 12
↓
Each group of 12 is made up of 2 groups of 6.



9×4
9 groups of 4



$9 \times (2 \times 2)$
9 groups of 4
↓
Each group of 4 is made up of 2 groups of 2.

Multiplying More than Two Numbers (page 2 of 2)

Math Words
- prime factorization

Here are ways to multiply whole numbers to make 36.

two factors	three factors	four factors
2 x 18	2 x 2 x 9	2 x 2 x 3 x 3
3 x 12	2 x 3 x 6	
4 x 9	3 x 3 x 4	
6 x 6		

$2 \times 2 \times 3 \times 3$ is the longest multiplication expression with a product of 36 using only whole numbers greater than 1.

(2) x (2) x (3) x (3)

Notice that these factors are prime numbers.

$2 \times 2 \times 3 \times 3$ is the prime factorization of 36.

Find the prime factorization of 120.

Multiplication Combinations

(page 1 of 5)

One of your goals in math class this year is to review and practice all the multiplication combinations up to 12 × 12.

1 x 1	1 x 2	1 x 3	1 x 4	1 x 5	1 x 6	1 x 7	1 x 8	1 x 9	1 x 10	1 x 11	1 x 12
2 x 1	2 x 2	2 x 3	2 x 4	2 x 5	2 x 6	2 x 7	2 x 8	2 x 9	2 x 10	2 x 11	2 x 12
3 x 1	3 x 2	3 x 3	3 x 4	3 x 5	3 x 6	3 x 7	3 x 8	3 x 9	3 x 10	3 x 11	3 x 12
4 x 1	4 x 2	4 x 3	4 x 4	4 x 5	4 x 6	4 x 7	4 x 8	4 x 9	4 x 10	4 x 11	4 x 12
5 x 1	5 x 2	5 x 3	5 x 4	5 x 5	5 x 6	5 x 7	5 x 8	5 x 9	5 x 10	5 x 11	5 x 12
6 x 1	6 x 2	6 x 3	6 x 4	6 x 5	6 x 6	6 x 7	6 x 8	6 x 9	6 x 10	6 x 11	6 x 12
7 x 1	7 x 2	7 x 3	7 x 4	7 x 5	7 x 6	7 x 7	7 x 8	7 x 9	7 x 10	7 x 11	7 x 12
8 x 1	8 x 2	8 x 3	8 x 4	8 x 5	8 x 6	8 x 7	8 x 8	8 x 9	8 x 10	8 x 11	8 x 12
9 x 1	9 x 2	9 x 3	9 x 4	9 x 5	9 x 6	9 x 7	9 x 8	9 x 9	9 x 10	9 x 11	9 x 12
10 x 1	10 x 2	10 x 3	10 x 4	10 x 5	10 x 6	10 x 7	10 x 8	10 x 9	10 x 10	10 x 11	10 x 12
11 x 1	11 x 2	11 x 3	11 x 4	11 x 5	11 x 6	11 x 7	11 x 8	11 x 9	11 x 10	11 x 11	11 x 12
12 x 1	12 x 2	12 x 3	12 x 4	12 x 5	12 x 6	12 x 7	12 x 8	12 x 9	12 x 10	12 x 11	12 x 12

There are 144 multiplication combinations on this chart. You may think that remembering all of them is a challenge, but you should not worry. On the next few pages you will find some suggestions for learning many of them.

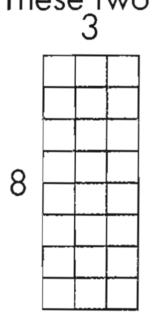
Multiplication Combinations

(page 2 of 5)

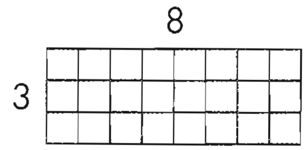
Learning Two Combinations at a Time

To help you review multiplication combinations, think about two combinations at a time, such as 8 × 3 and 3 × 8.

These two problems look different but have the same answer.



8 × 3



3 × 8

When you know that 8 × 3 = 24, you also know that 3 × 8 = 24.

You've learned two multiplication combinations!

By "turning around" combinations and learning them two at a time, the chart of multiplication combinations is reduced from 144 to 78 combinations to learn.

1 x 1	1 x 2	1 x 3	1 x 4	1 x 5	1 x 6	1 x 7	1 x 8	1 x 9	1 x 10	1 x 11	1 x 12
2 x 1 1 x 2	2 x 2	2 x 3	2 x 4	2 x 5	2 x 6	2 x 7	2 x 8	2 x 9	2 x 10	2 x 11	2 x 12
3 x 1 1 x 3	3 x 2 2 x 3	3 x 3	3 x 4	3 x 5	3 x 6	3 x 7	3 x 8	3 x 9	3 x 10	3 x 11	3 x 12
4 x 1 1 x 4	4 x 2 2 x 4	4 x 3 3 x 4	4 x 4	4 x 5	4 x 6	4 x 7	4 x 8	4 x 9	4 x 10	4 x 11	4 x 12
5 x 1 1 x 5	5 x 2 2 x 5	5 x 3 3 x 5	5 x 4 4 x 5	5 x 5	5 x 6	5 x 7	5 x 8	5 x 9	5 x 10	5 x 11	5 x 12
6 x 1 1 x 6	6 x 2 2 x 6	6 x 3 3 x 6	6 x 4 4 x 6	6 x 5 5 x 6	6 x 6	6 x 7	6 x 8	6 x 9	6 x 10	6 x 11	6 x 12
7 x 1 1 x 7	7 x 2 2 x 7	7 x 3 3 x 7	7 x 4 4 x 7	7 x 5 5 x 7	7 x 6 6 x 7	7 x 7	7 x 8	7 x 9	7 x 10	7 x 11	7 x 12
8 x 1 1 x 8	8 x 2 2 x 8	8 x 3 3 x 8	8 x 4 4 x 8	8 x 5 5 x 8	8 x 6 6 x 8	8 x 7 7 x 8	8 x 8	8 x 9	8 x 10	8 x 11	8 x 12
9 x 1 1 x 9	9 x 2 2 x 9	9 x 3 3 x 9	9 x 4 4 x 9	9 x 5 5 x 9	9 x 6 6 x 9	9 x 7 7 x 9	9 x 8 8 x 9	9 x 9	9 x 10	9 x 11	9 x 12
10 x 1 1 x 10	10 x 2 2 x 10	10 x 3 3 x 10	10 x 4 4 x 10	10 x 5 5 x 10	10 x 6 6 x 10	10 x 7 7 x 10	10 x 8 8 x 10	10 x 9 9 x 10	10 x 10	10 x 11	10 x 12
11 x 1 1 x 11	11 x 2 2 x 11	11 x 3 3 x 11	11 x 4 4 x 11	11 x 5 5 x 11	11 x 6 6 x 11	11 x 7 7 x 11	11 x 8 8 x 11	11 x 9 9 x 11	11 x 10 10 x 11	11 x 11	11 x 12
12 x 1 1 x 12	12 x 2 2 x 12	12 x 3 3 x 12	12 x 4 4 x 12	12 x 5 5 x 12	12 x 6 6 x 12	12 x 7 7 x 12	12 x 8 8 x 12	12 x 9 9 x 12	12 x 10 10 x 12	12 x 11 11 x 12	12 x 12

Multiplication Combinations

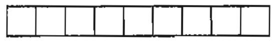
(page 3 of 5)

Another helpful way to learn multiplication combinations is to think about one category at a time. Here are some categories you may have seen before.

Learning the ×1 Combinations

You may be thinking about only one group.

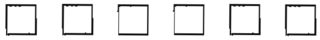
1 group of 9 equals 9.



→ 1 x 9 = 9

You may also be thinking about several groups of 1.

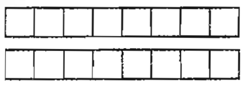
6 groups of 1 equal 6.



→ 6 x 1 = 6

Learning the ×2 Combinations

Multiplying by 2 is the same as doubling a number.



→ 8 + 8 = 16

→ 2 x 8 = 16

Learning the ×10 and ×5 Combinations

You can learn these combinations by skip counting by 10s and 5s.

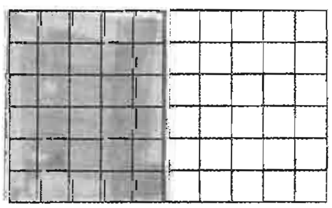
10, 20, 30, 40, 50, 60 → 6 x 10 = 60

5, 10, 15, 20, 25, 30 → 6 x 5 = 30

Another way to find a ×5 combination is to remember that it is half of a ×10 combination.

10

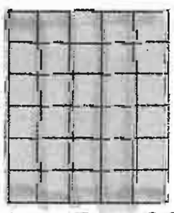
6



6 × 10 = 60

5

6



6 × 5 = 30

6 x 5 (or 30) is half of 6 x 10 (or 60).

Multiplication Combinations

(page 4 of 5)

Here are some more categories to help you learn the multiplication combinations.

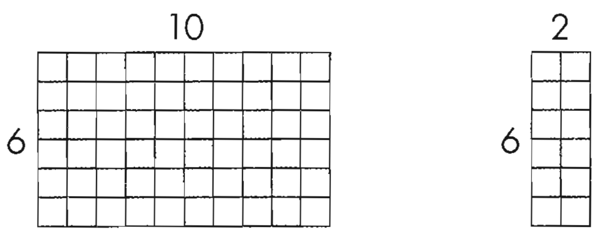
Learning the ×11 Combinations

Many students learn these combinations by noticing the double-digit pattern they create.

$11 \times 3 = 33$ $11 \times 4 = 44$ $11 \times 5 = 55$ $11 \times 6 = 66$ $11 \times 7 = 77$

Learning the ×12 Combinations

Many students multiply by 12 by breaking the 12 into 10 and 2.



6 × 10 = 60   *6 × 2 = 12*

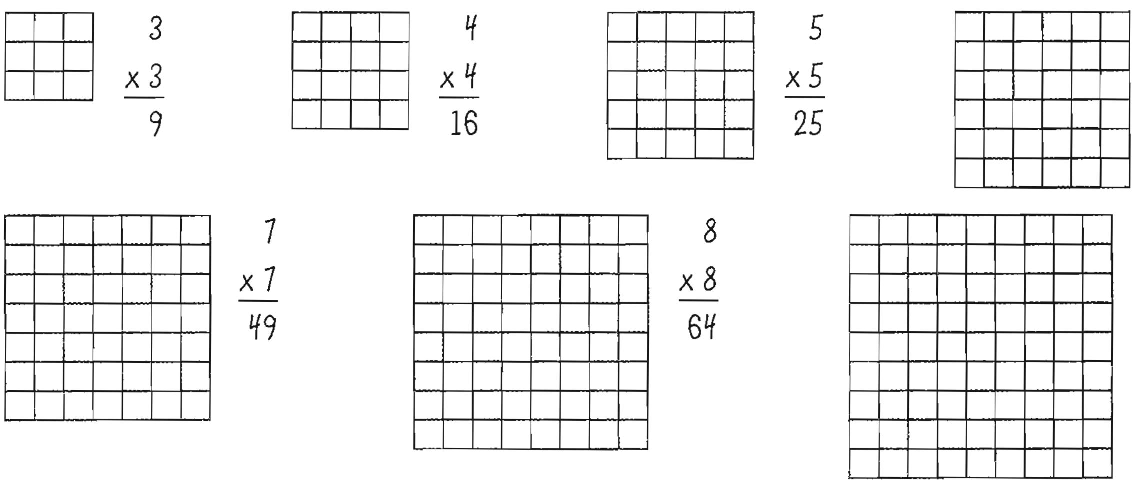
$6 \times 12 = (6 \times 10) + (6 \times 2)$

$6 \times 12 = 60 + 12$

$6 \times 12 = 72$

Learning the Square Numbers

Many students remember the square number combinations by building the squares with tiles or drawing them on grid paper.



$3 \times 3 = 9$ $4 \times 4 = 16$ $5 \times 5 = 25$ $6 \times 6 = 36$

$7 \times 7 = 49$ $8 \times 8 = 64$ $9 \times 9 = 81$

Multiplication Combinations

(page 5 of 5)

After you have used all these categories to practice the multiplication combinations, you have only a few more to learn.

1 x 1	1 x 2	1 x 3	1 x 4	1 x 5	1 x 6	1 x 7	1 x 8	1 x 9	1 x 10	1 x 11	1 x 12
2 x 1	2 x 2	2 x 3	2 x 4	2 x 5	2 x 6	2 x 7	2 x 8	2 x 9	2 x 10	2 x 11	2 x 12
3 x 1	3 x 2	3 x 3	3 x 4	3 x 5	3 x 6	3 x 7	3 x 8	3 x 9	3 x 10	3 x 11	3 x 12
4 x 1	4 x 2	4 x 3 3 x 4	4 x 4	4 x 5	4 x 6	4 x 7	4 x 8	4 x 9	4 x 10	4 x 11	4 x 12
5 x 1	5 x 2	5 x 3	5 x 4	5 x 5	5 x 6	5 x 7	5 x 8	5 x 9	5 x 10	5 x 11	5 x 12
6 x 1	6 x 2	6 x 3 3 x 6	6 x 4 4 x 6	6 x 5	6 x 6	6 x 7	6 x 8	6 x 9	6 x 10	6 x 11	6 x 12
7 x 1	7 x 2	7 x 3 3 x 7	7 x 4 4 x 7	7 x 5	7 x 6 6 x 7	7 x 7	7 x 8	7 x 9	7 x 10	7 x 11	7 x 12
8 x 1	8 x 2	8 x 3 3 x 8	8 x 4 4 x 8	8 x 5	8 x 6 6 x 8	8 x 7 7 x 8	8 x 8	8 x 9	8 x 10	8 x 11	8 x 12
9 x 1	9 x 2	9 x 3 3 x 9	9 x 4 4 x 9	9 x 5	9 x 6 6 x 9	9 x 7 7 x 9	9 x 8 8 x 9	9 x 9	9 x 10	9 x 11	9 x 12
10 x 1	10 x 2	10 x 3	10 x 4	10 x 5	10 x 6	10 x 7	10 x 8	10 x 9	10 x 10	10 x 11	10 x 12
11 x 1	11 x 2	11 x 3	11 x 4	11 x 5	11 x 6	11 x 7	11 x 8	11 x 9	11 x 10	11 x 11	11 x 12
12 x 1	12 x 2	12 x 3	12 x 4	12 x 5	12 x 6	12 x 7	12 x 8	12 x 9	12 x 10	12 x 11	12 x 12

As you practice all of the multiplication combinations, there will be some that you "just know" and others that you are "working on" learning. To practice the combinations that are difficult for you to remember, think of a combination that you know as a clue to help you. Here are some suggestions.

$9 \times 8 = 72$ $8 \times 9 = 72$	Clue:	10 x 8 = 80	80 − 8 = 72	
$6 \times 7 = 42$ $7 \times 6 = 42$	Clue:	6 x 5 = 30	6 x 2 = 12	30 + 12 = 42
$4 \times 8 = 32$ $8 \times 4 = 32$	Clue:	2 x 8 = 16	16 + 16 = 32	

22

Multiplication Strategies (page 1 of 3)

In Grade 5, you are learning how to solve multiplication problems efficiently.

There are 38 rows in an auditorium, and 26 chairs in each row. How many people can sit in the auditorium?

Breaking the Numbers Apart

Georgia solved the problem 38×26 by breaking apart both factors.

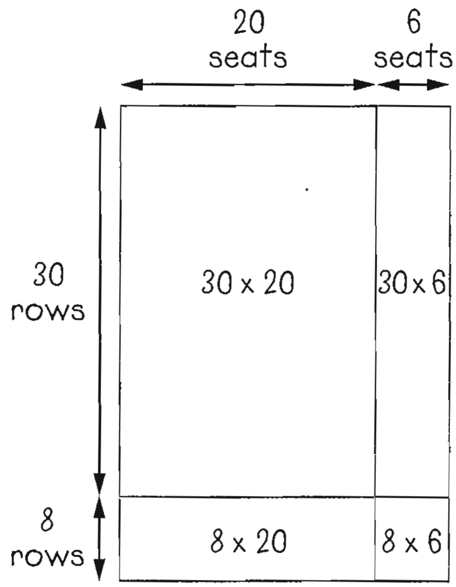
Georgia's solution

First I'll figure out how many people are in the first 30 rows.

$30 \times 20 = 600$	*That's the first 30 rows, with 20 people in each row.*
$30 \times 6 = 180$	*That's 6 more people in each of those 30 rows, so now I've filled up 30 rows.*

There are 8 more rows to fill.

$8 \times 20 = 160$	*That's 20 people in those last 8 rows.*
$8 \times 6 = 48$	*I've filled up the last 8 rows with 6 more people in each row.*



Now I add together all the parts I figured out to get the answer.

$600 + 180 + 160 + 48 = \mathbf{988}$

988 people can sit in the auditorium.

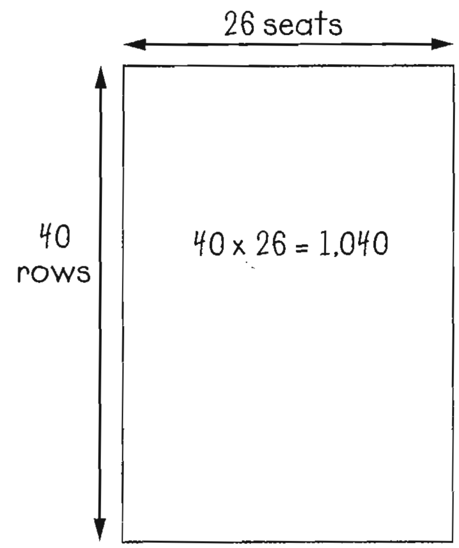
Solve 14×24 by using this first step: $14 \times 20 =$ ___?___

Multiplication Strategies (page 2 of 3)

There are 38 rows in the auditorium, and 26 chairs in each row. How many people can sit in the auditorium?

Changing One Number to Make an Easier Problem

Benson solved the auditorium problem, 38 × 26, by changing the 38 to 40 to make an easier problem.

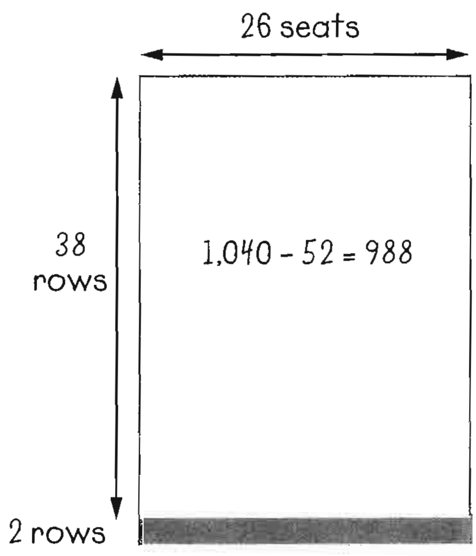


Benson's solution

I'll pretend that there are 40 rows in the auditorium instead of 38.

40 x 26 = 1,040	*I knew that 10 × 26 = 260. I doubled that to get 520, and doubled that to get 1,040.*

So, if there were 40 rows, 1,040 people could sit in the auditorium. But there are really only 38 rows, so I have 2 extra rows of 26 chairs. I need to subtract those.



2 x 26 = 52	*I need to subtract 52. I'll do that in two parts.*
1,040 − 40 = 1,000	*First I'll subtract 40.*
1,000 − 12 = 988	*Then I'll subtract 12.*

So, **988** people can sit in the auditorium.

Solve 19 × 14 by using this first step: 20 × 14 = ?

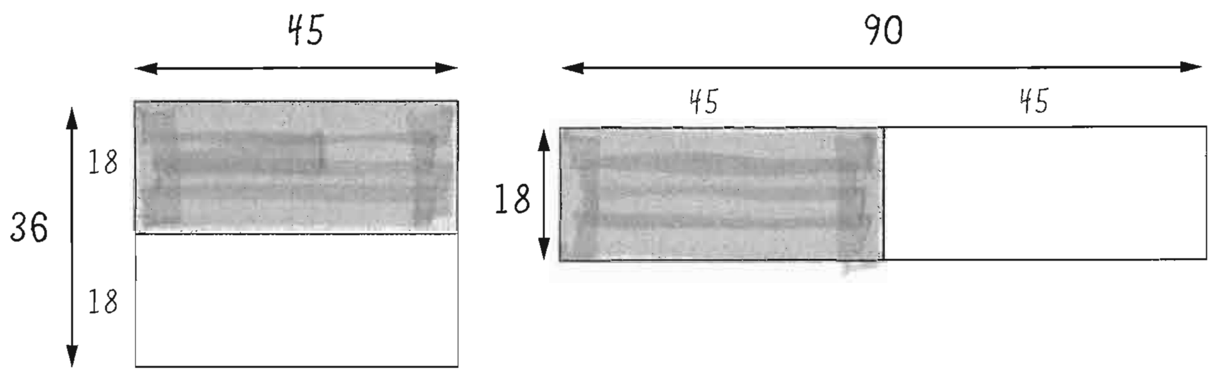
Multiplication Strategies (page 3 of 3)

A classroom measures 36 feet by 45 feet. How many 1-foot-square tiles will cover the floor?

Creating an Equivalent Problem

Nora's solution

I can double 45 and take half of 36 and pretend to change the shape of the classroom.



A 36-foot by 45-foot classroom needs the same amount of floor tiles as a 18-foot by 90-foot classroom.

For me, 18×90 is an easier problem to solve.

$10 \times 90 = 900$

$8 \times 90 = 720$

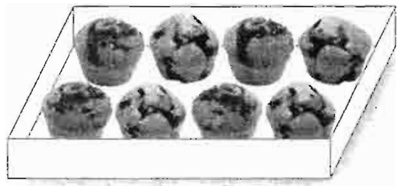
$18 \times 90 = \mathbf{1{,}620}$

1,620 tiles will cover the floor.

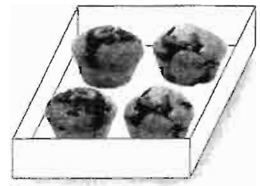
Solve: $35 \times 22 = \underline{\ ?\ } \times 11$

Equivalent Expressions in Multiplication (page 1 of 2)

A large box holds twice as many muffins as a small box.

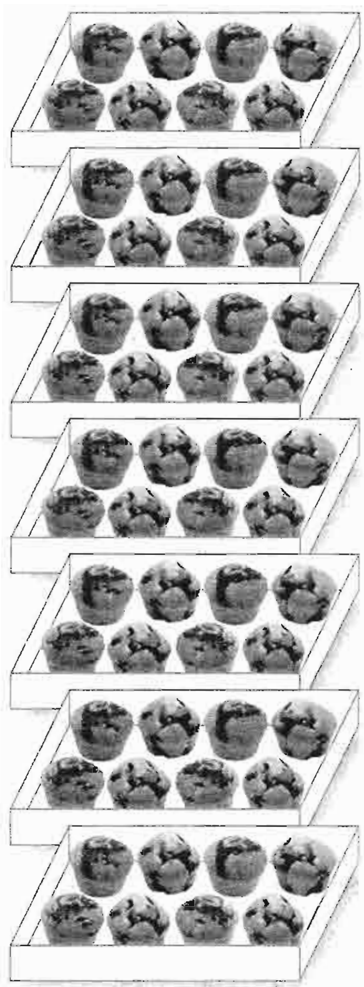


large box

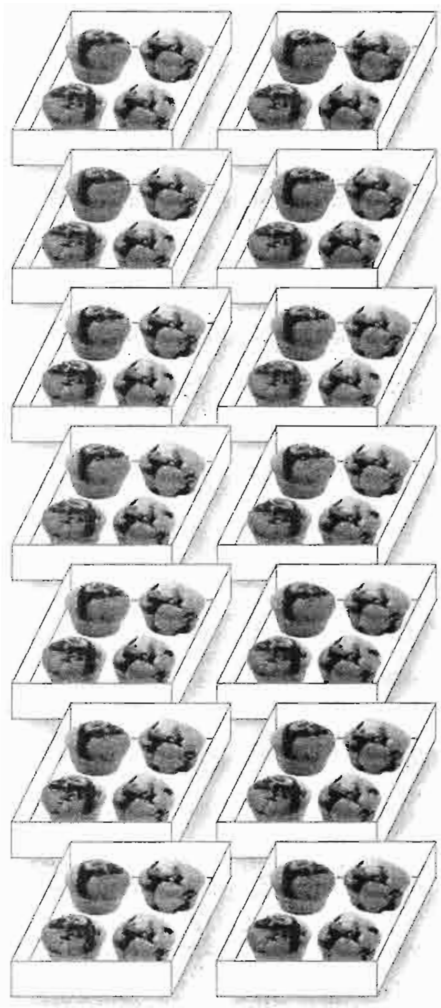


small box

A customer ordered 7 large boxes of muffins at the bakery. The baker only had small boxes. How many small boxes of muffins should the customer buy to get the same number of muffins?



7 boxes with 8 muffins in each box



14 boxes with 4 muffins in each box

The small boxes are half the size of the large boxes. The customer should buy twice as many small boxes.

double (7 → 14)

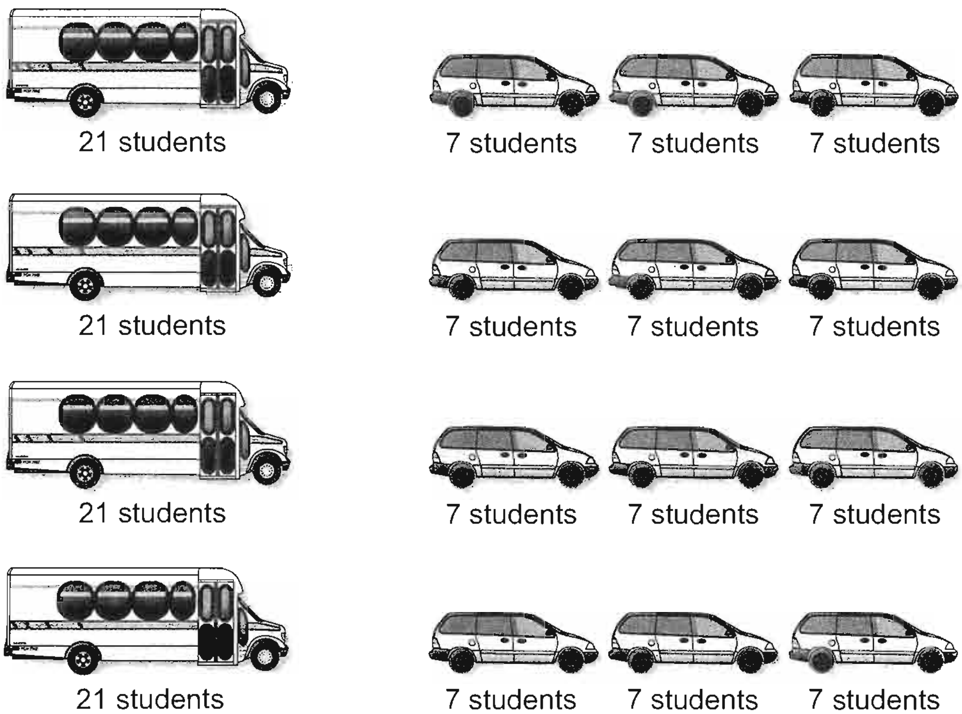
$$7 \times 8 = 14 \times 4$$

half (8 → 4)

Equivalent Expressions in Multiplication (page 2 of 2)

The fifth grade is going on a field trip. The teachers planned to take 4 buses. Instead they need to take vans. How many vans do they need?

A bus holds three times as many students as a van.



four buses with 21 students in each bus

twelve vans with 7 students in each van

The vans hold one third as many students as the buses do. The teachers need three times as many vans.

triple

$$4 \times 21 = 12 \times 7$$

third

Create an equivalent problem:
4 × 12 = ______ × ______

SMH

our

Multiplication and Division Cluster Problems

Cluster problems help you use what you know about easier problems to solve harder problems.

1. Solve the problems in each cluster.
2. Use one or more of the problems in the cluster to solve the final problem, along with other problems if you need them.

Solve these cluster problems:	How did you solve the final problem?
$24 \times 10 =$ 240 $24 \times 3 =$ 72 $24 \times 20 =$ 480 $24 \times 30 =$ 720 Now solve this problem: $24 \times 31 =$ **744**	*I figured out that 24×30 would be 720 because $24 \times 10 = 240$, and $240 + 240 + 240 = 720$.* *I need one more group of 24. That's $720 + 24 = 744$. So, $24 \times 31 = 744$.*

Solve these cluster problems:	How did you solve the final problem?
$10 \times 12 =$ 120 $5 \times 12 =$ 60 Now solve this problem: $192 \div 12 =$ **16**	*I thought of $192 \div 12$ as ____ $\times 12 = 192$. $10 \times 12 = 120$ and $5 \times 12 = 60$, so $15 \times 12 = 120 + 60 = 180$.* *I need one more 12 to get to 192. $16 \times 12 = 192$ So, $192 \div 12 = 16$.*

Solve these cluster problems:	How did you solve the final problem?
$54 \div 6 =$ 9 $540 \div 6 =$ 90 Now solve this problem: $6\overline{)546}$ **91**	*After I knew $540 \div 6 = 90$, then I knew I needed one more group of 6 because $546 = 540 + 6$.* *So, $546 \div 6 = 91$.*

Comparing Multiplication Algorithms

Math Words
- **algorithm**

Some fifth graders compared these two algorithms.

An algorithm is a step-by-step procedure to solve a certain kind of problem.

Partial Products

		278
		x 35
(5 x 8)	→	40
(5 x 70)	→	350
(5 x 200)	→	1,000
(30 x 8)	→	240
(30 x 70)	→	2,100
(30 x 200)	→	6,000
		9,730

U.S. Algorithm

2 2 3 4 278		
x 35		
1,390	←	(5 x 278)
8,340	←	(30 x 278)
9,730		

Here are some of the things the students noticed.

- *Both solutions involve breaking apart numbers.*
- *The first three numbers in the partial products algorithm are combined in the first number in the solution using the U.S. algorithm.*
- *The algorithms are mostly the same, but the U.S. algorithm notation combines steps (40 + 350 + 1,000 = 1,390).*
- *The little numbers in the U.S. algorithm stand for tens and hundreds. The 4 and the 2 above the 7 are really 40 and 20.*

Multiplication and Division Cluster Problems

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1. Solve the problems in each cluster.
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Solve these cluster problems:	How did you solve the final problem?
$54 \div 6 =$ 9 $540 \div 6 =$ 90 Now solve this problem: $6\overline{)546}$ **91**	*After I knew $540 \div 6 = 90$, then I knew I needed one more group of 6 because $546 = 540 + 6$.* *So, $546 \div 6 = 91$.*

Remainders: What Do You Do with the Extras?

Math Words
- **remainder**

When you are asked to solve division problems in context, it is important to consider the remainder to correctly answer the question asked. Here are some different story problem contexts for the division problem 186 ÷ 12 = 15 R6.

186 people are taking a trip. One van holds 12 people. How many vans do they need?

15 vans will hold 15 × 12 or 180 people, but the other 6 people still need a ride. They need 1 more van.

Answer: **They need 16 vans.**

There are 186 pencils and 12 students. A teacher wants to give the same number of pencils to each student. How many pencils will each student get?

It does not make sense to give students half a pencil, so the teacher can keep the remaining 6 pencils.

Answer: **Each student will get 15 pencils.**

Twelve friends earned $186 by washing cars. They want to share the money equally. How much money should each person get?

Dollars can be split up into smaller amounts. Each person can get $15. The remaining $6 can be divided evenly so that every person gets another 50¢.

Answer: **Each person gets $15.50.**

Twelve people are going to share 186 crackers evenly. How many crackers does each person get?

Each person gets 15 crackers. Then the last 6 crackers can be split in half. Each person gets another half cracker.

Answer: **Each person gets $15\frac{1}{2}$ crackers.**

Write and solve a story problem for 153 ÷ 13.

Division Strategies (page 1 of 2)

In Grade 5, you are learning how to solve division problems efficiently.

Here is an example of a division problem.

Janet has 1,780 marbles. She wants to put them into bags, each of which holds 32 marbles. How many full bags of marbles will she have?

Samantha solved this problem by multiplying groups of 32 to reach 1,780.

Samantha's solution

30	x	32	=	960	There are 960 marbles in 30 bags of 32.
20	x	32	=	640	There are 640 marbles in 20 bags of 32.
5	x	32	=	160	There are 160 marbles in 5 bags of 32.
55				1,760	There are 1,760 marbles in 55 bags of 32.

1,760 is as close as I can get to 1,780 with groups of 32.

$1{,}780 \div 32 = 55$ R20

Janet can fill 55 bags, and she will have 20 extra marbles.

Talisha solved this problem by subtracting groups of 32 from 1,780.

Talisha's solution

32)1,780		
−640	20 bags	
1,140		
−640	20 bags	
500		→ **55 bags**
−320	10 bags	
180		
−160	5 bags	
20	→ **20 extra marbles**	

Division Strategies (page 2 of 2)

Here is another division example.

54)2,500

Hana solved this problem by subtracting groups of 54 from 2,500.

Hana's solution

```
54)2,500
  - 1,080  (20)
    1,420
  - 1,080  (20)
      340
    - 216   (4)
      124
    - 108   (2)
       16   46 R16
```

Walter solved this problem by multiplying groups of 54 to reach 2,500.

Walter's solution

```
 10  x 54 =   540
 20  x 54 = 1,080
(40) x 54 = 2,160 → 2,160
(4)  x 54 =   216 →   216
(1)  x 54 =    54 →    54
(1)  x 54 =    54 →    54
                    2,484
```

2,500 ÷ 54 = **46 R16**

How would you solve this problem? 54)2,500

Dividing with 2 digit divisors

Chunking/Clustering

$609 \div 29$

$290 + 290 + 29 = 609$

$10 + 10 + 1 = 21$

$840 \div 32$

$320 + 320 + 160 + 32 + 8 = 840$

$10 + 10 + 5 + 1 \text{ R}8 = 26 \text{ R}8$

Partial Quotient

29) 609	
−290	10
319	
−290	10
29	
−29	1
0	21

32) 840	
−320	10
520	
−320	10
200	
−160	5
40	
−32	1
8	26 R8

Area Model

	20	+ 1
10	200	10
10	200	10
5	100	5
4	80	4

= 21

+ 29 * 580 + 29 = 609

	20	+ 5	+ 1	
10	200	50	10	
10	200	50	10	
10	200	50	10	
+ 2	40	10	2	8

= 26 R8

32 640 + 160 32

800 + 32 = 832 + 8 = 840

Ratio table

1	2	10	20
29	58	290	580

20+1 = 21

580 + 29 = 609

1	2	4	10	20	40
32	64	128	320	640	1280

= 20+4+2 = 26 R8

640 + 128 = 768 + 64 = 832 + 8

Chunking

615 ÷ 5

500 50 50 15

100 + 10 + 10 + 3 = 123

Partial Quotient

```
5 | 615
    500  | 100
    115
  -  50  | 10
     65
     50  | 10
     15
   - 15  | 3
      0   (123)
```

Area Model

	100	10	10	3
5	500	50	50	15

= 123

Ratio Table

1	10	20	100	5
5	50	100	500	25

Chunking 982 ÷ 2

200 200 200 200 100 80 2

800 + 100 + 80 + 2 =

100 + 100 + 100 + 100 + 50 + 40 + 1 = 491

Partial Product

2) 982	
− 200	100
782	
200	100
582	
− 200	100
382	
− 200	100
182	
100	50
82	
− 80	40
2	
− 2	1
0	491

96 ÷ 4

Chunking

40 + 40 + 4 + 4 + 4 + 4 = 96

= 10 + 10 + 1 + 1 + 1 + 1 = (24)

Partial Quotient

```
4 | 96
  - 40    10
    56
  - 40    10
    16
  - 16     4
     0
         (24)
```

Area Model

10 + 10 + 4 = (24)

4	40	40	16

Ratio Table

1	10	20	5	4
4	40	80	20	16

80 + 16 = 96

(20 + 4) = (24)

Chunking

$70 \div 5$

50 + 20

10 + 4 = (14)

Partial Quotient

$$
\begin{array}{r|l}
5\,\overline{)70} & \\
-50 & 10 \\
\hline
20 & \\
-10 & 2 \\
\hline
10 & \\
-10 & 2 \\
\hline
0 & \\
 & (14)
\end{array}
$$

Area Model

10 + 4 = (14)

5	50	20

1	10	5	2
5	50	25	10

10 + 2 + 2 = (14)

Fractions, Decimals, and Percents (page 1 of 2)

Math Words
- **fraction**
- **decimal**
- **percent**

Fractions, decimals, and percents are numbers that can be used to show parts of a whole.

fraction

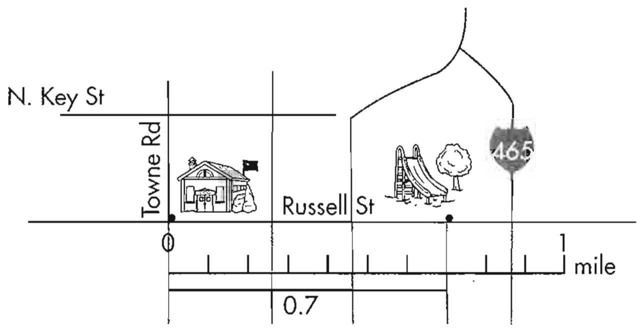
$\frac{5}{8}$

part: 5 striped T-shirts

whole: 8 T-shirts in the group

decimal

0.7



part: $\frac{7}{10}$ of a mile from school to the park

whole: 1 mile

percent

90%

SPELLING TEST 90%
1. geometry
2. quadrilaterall
3. trapezoid
4. rhombus
5. pentagon
6. hexagon
7. octagon
8. isosceles
9. obtuse
10. acute

part: 9 words spelled correctly

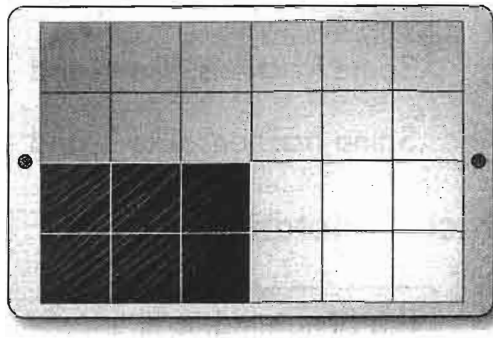
whole: 10 words on the spelling test

Look in magazines and newspapers to find more everyday uses of fractions, decimals and percents.

SMH

Fractions, Decimals, and Percents (page 2 of 2)

These questions can be answered using fractions, decimals or percents.



How much of the batch of brownies is left in the pan?

6 out of 24 brownies

We would most likely say:

$\frac{1}{4}$ *of the brownies are in the pan.*

It would also be correct to say:

25% of the brownies are in the pan.

0.25 of the brownies are in the pan.

These values are equivalent because each one represents the same quantity.

$\frac{6}{24} = \frac{1}{4} = 25\% = 0.25$

At the softball game, Nora went to bat 6 times and got a hit 3 of those 6 times.

Nora got a hit $\frac{1}{2}$ *of the time she went to bat.*

Nora got a hit 50% of the time she went to bat.

Nora's batting average for the game was .500.

These values are equivalent because each one represents the same quantity.

$\frac{3}{6} = \frac{1}{2} = 50\% = 0.500$

Fractions

Math Words
- **fraction**
- **numerator**
- **denominator**

Fractions are numbers.

Some fractions, like $\frac{1}{2}$ and $\frac{3}{4}$, are less than 1.

Some fractions, like $\frac{2}{2}$ and $\frac{4}{4}$, are equal to 1.

Some fractions, like $\frac{6}{4}$ and $\frac{3}{2}$, are greater than 1.

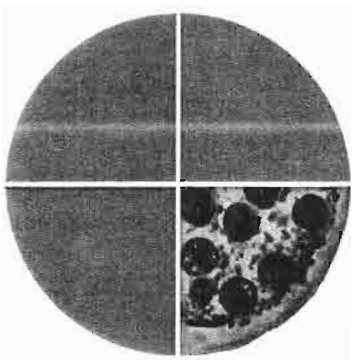
Fraction Notation

$\frac{3}{4}$ three fourths

The denominator is the total number of equal shares. → 4

The numerator is the number of equal shares out of the total. → 3

Charles cut a pizza into four equal pieces and ate three pieces.



$\frac{3}{4}$

The whole pizza has four equal parts, or slices. → 4

Charles ate three pieces. → 3

3 out of 4 equal pieces were eaten.

Samantha has 12 marbles in her collection. Nine twelfths of her marbles are blue.

$\frac{9}{12}$

There are 12 marbles in the whole group. → 12

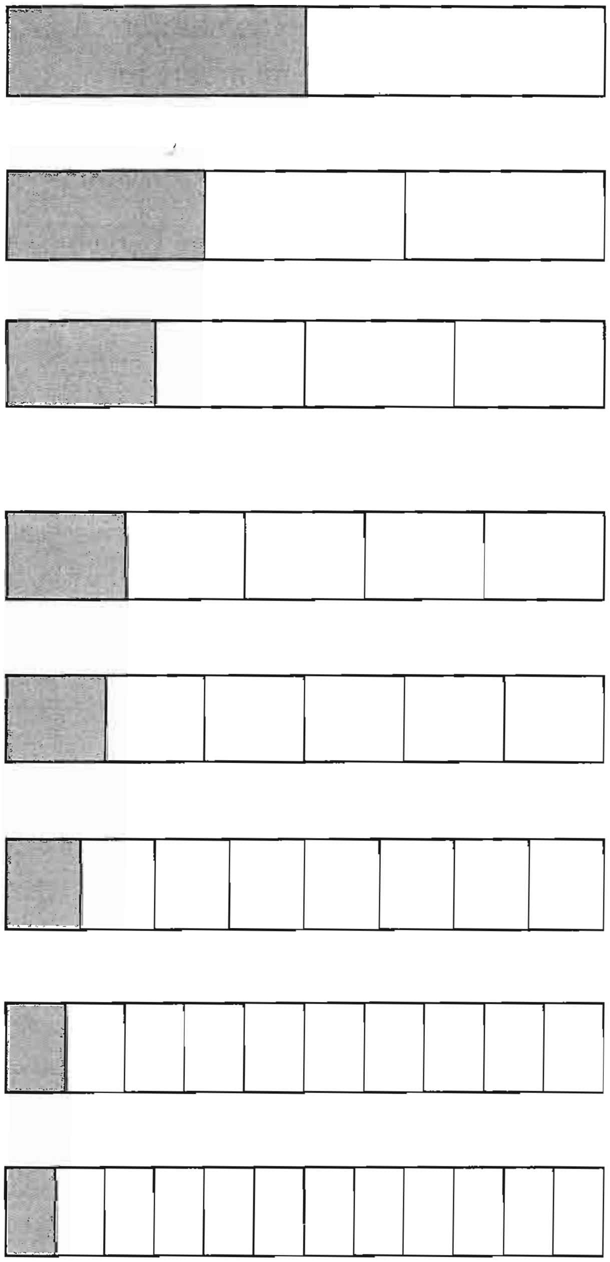
Nine of the marbles are blue. → 9

9 out of 12 equal parts are blue.

What fraction of the pizza was not eaten?
What fraction of the marbles are not blue?

Naming Fractions

In each of these examples, one whole rectangle has been divided into equal parts.



$\frac{1}{2}$ one half green	$\frac{1}{2}$ one half white
$\frac{1}{3}$ one third green	$\frac{2}{3}$ two thirds white
$\frac{1}{4}$ one fourth (one quarter) green	$\frac{3}{4}$ three fourths (three quarters) white
$\frac{1}{5}$ one fifth green	$\frac{4}{5}$ four fifths white
$\frac{1}{6}$ one sixth green	$\frac{5}{6}$ five sixths white
$\frac{1}{8}$ one eighth green	$\frac{7}{8}$ seven eighths white
$\frac{1}{10}$ one tenth green	$\frac{9}{10}$ nine tenths white
$\frac{1}{12}$ one twelfth green	$\frac{11}{12}$ eleven twelfths white

It's interesting that, out of all of these examples, 12 is the biggest number of parts, but that rectangle has the smallest parts.

Using Fractions for Quantities Greater Than One

Math Words

- **mixed number**

To represent fractions greater than one, you need more than one whole.

All of these boards are the same size.

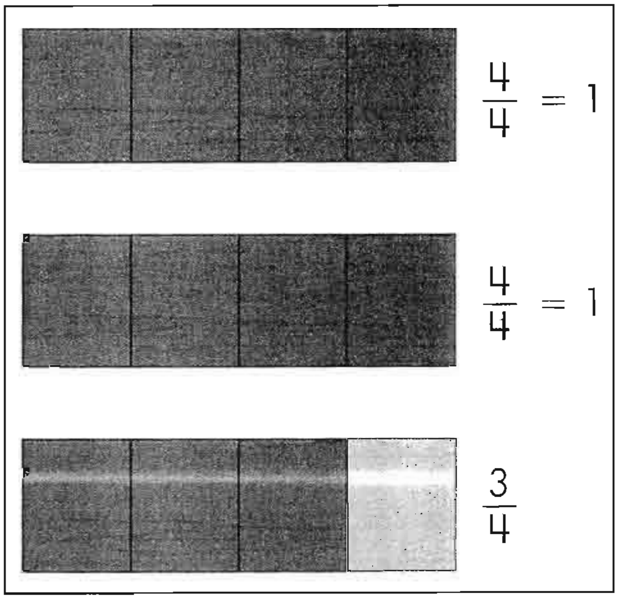
Each board is divided into 4 equal parts.

The first two whole boards are painted orange. The orange part is $\frac{8}{4}$, or 2.

On the last board, three parts are painted orange. The orange part of this board is $\frac{3}{4}$.

The total amount painted orange is $\frac{11}{4}$, or $2\frac{3}{4}$.

$$\frac{4}{4} + \frac{4}{4} + \frac{3}{4} = \frac{11}{4} = 2\frac{3}{4}$$

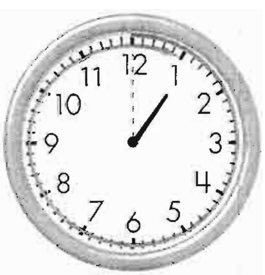


A mixed number has a whole number part and a fractional part.

whole number → $2\frac{3}{4}$ ← fraction

two and three fourths

Here is another example that uses a clock as a model.



The hour hand started at 12. It made one full rotation and then moved one more hour. The total rotation is $1\frac{1}{12}$, or $\frac{13}{12}$ of the way around the clock.

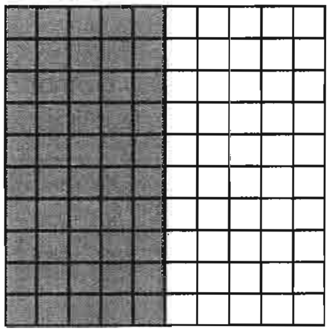
How can you represent these fractions? $\frac{5}{3}$ $1\frac{1}{6}$

Percents

Math Words
- **percent**

Percent means "out of 100" or "hundredths."

Fifty percent of this 10 × 10 square is shaded.



percent symbol

50%

50 out of 100

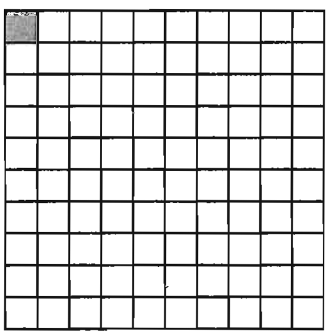
Every percent can be written as a decimal, using hundredths.

Every percent can be written as a fraction with 100 in the denominator.

$$50\% = 0.50 = 0.5 = \frac{50}{100} = \frac{1}{2}$$

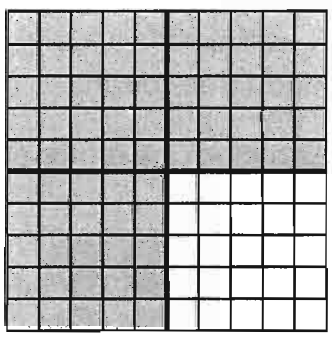
Percents can also be written as other equivalent fractions and decimals.

Here are some other examples.



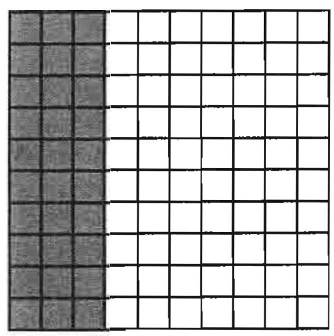
1 out of 100

$1\% = 0.01 = \frac{1}{100}$



75 out of 100

$75\% = 0.75 = \frac{75}{100} = \frac{3}{4}$



30 out of 100

$30\% = 0.30 = 0.3 = \frac{30}{100} = \frac{3}{10}$

Decimals

Math Words
- **decimal**

The system we use to write numbers is called the decimal number system. *Decimal* means that the number is based on tens.

Some numbers, like 2.5 and 0.3, include a decimal point. The digits to the right of the decimal point are part of the number that is less than 1.

Here are some examples you may know of decimal numbers that are less than one.

$0.5 = \frac{5}{10} = \frac{1}{2}$	$0.25 = \frac{25}{100} = \frac{1}{4}$

Numbers such as 0.5 and 0.25 are sometimes called decimal fractions.

Some decimal numbers have a whole number part and a part that is less than 1, just as mixed numbers do.

$1.5 = 1\frac{5}{10} = 1\frac{1}{2}$	$12.75 = 12\frac{75}{100} = 12\frac{3}{4}$

Here are some examples of the ways we use decimals everyday:

Total rainfall in the last 24 hours: 0.25 inch

The race is a little more than 26 miles.

She swam the race in a little less than 31 seconds.

Write a decimal number that is . . . a little more than 12.
. . . almost 6.
. . . more than $\frac{3}{4}$ and less than 1.

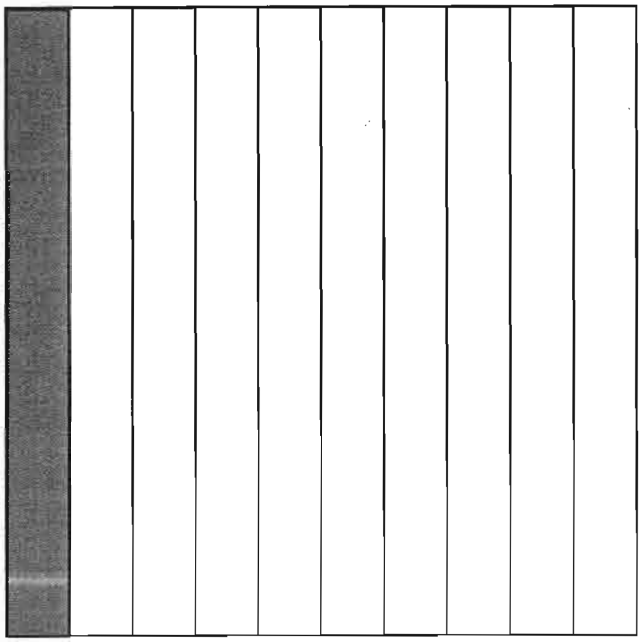
SMH

Representing Decimals

(page 1 of 2)

Math Words
- **tenths**
- **hundredths**

In each of the following examples, the whole square has been divided into equal parts and the amount shaded is named.

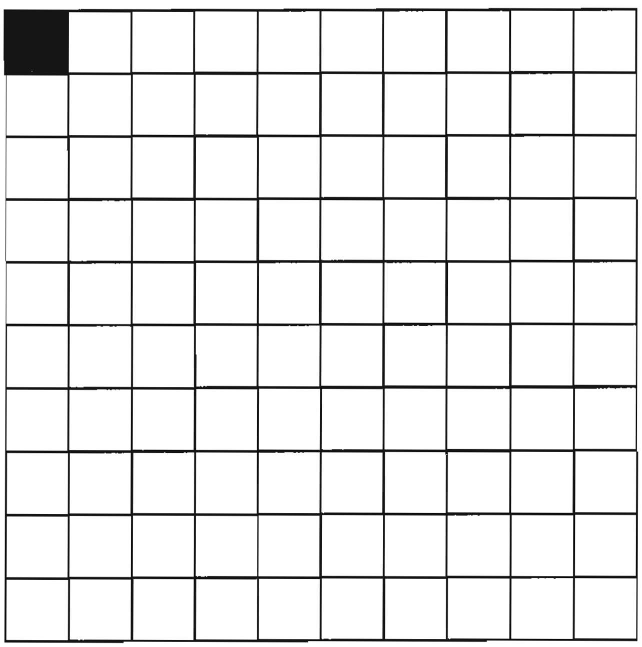


This square is divided into 10 parts. One out of the ten parts is shaded. Amount shaded:

one tenth

fraction: $\frac{1}{10}$

decimal: 0.1



This square is divided into 100 parts. One out of the hundred parts is shaded. Amount shaded:

one hundredth

fraction: $\frac{1}{100}$

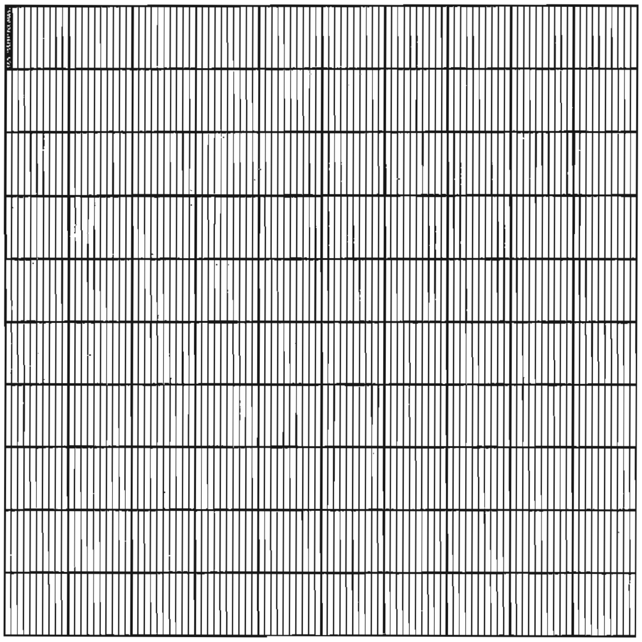
decimal: 0.01

Representing Decimals

(page 2 of 2)

Math Words
- **thousandths**
- **ten thousandths**

In each of the following examples, the whole square has been divided into equal parts and the amount shaded is named.

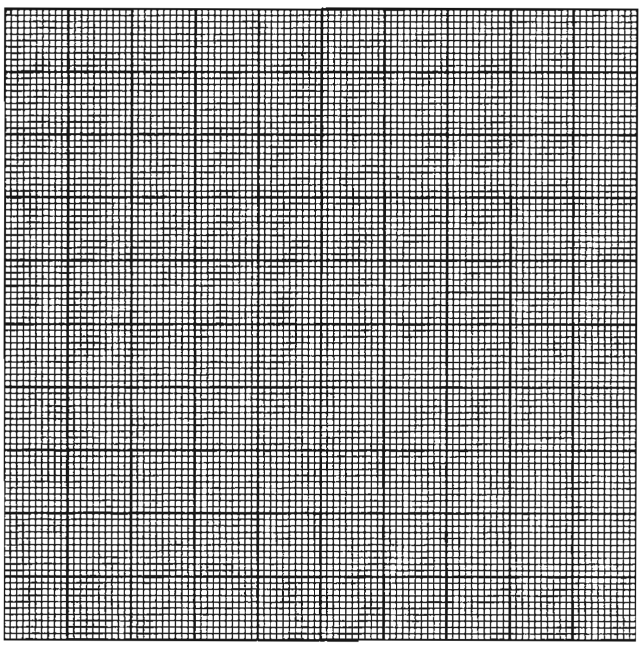


This square is divided into 1,000 parts. One out of the thousand parts is shaded. Amount shaded:

one thousandth

fraction: $\frac{1}{1000}$

decimal: 0.001



This square is divided into 10,000 parts. One out of the ten thousand parts is shaded. Amount shaded:

one ten-thousandth

fraction: $\frac{1}{10000}$

decimal: 0.0001

Can you prove that the thousandths square is divided into one thousand parts without counting them?

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Place Value of Decimals

Math Words
- **decimal point**

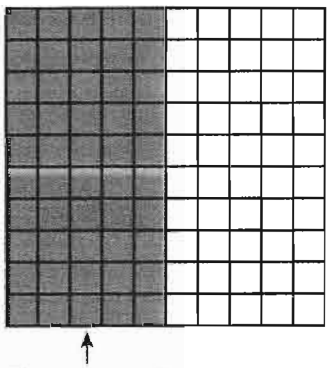
As with whole numbers, the value of a digit changes depending on its place in a decimal number.

thousands place , hundreds place | tens place | ones place | **.** | tenths place | hundredths place | thousandths place

decimal point

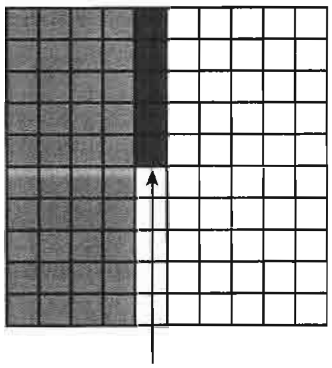
In these three examples the digit 5 has different **values:**

0.5



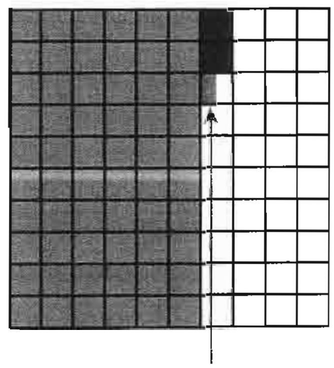
The digit 5 in the **tenths place** represents $\frac{5}{10}$.

0.45



The digit 5 in the **hundredths place** represents $\frac{5}{100}$.

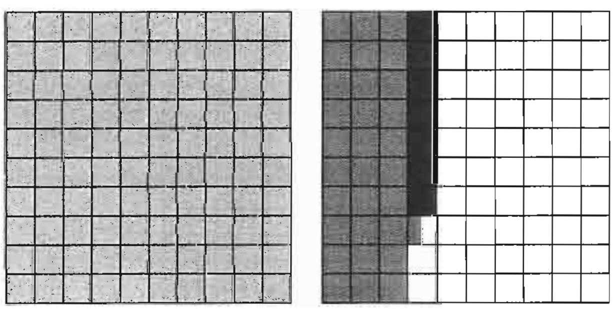
0.625



The digit 5 in the **thousandths place** represents $\frac{5}{1{,}000}$.

Look at the values of the digits in this number:

1.375 (one and three hundred seventy-five thousandths or $1\frac{375}{1{,}000}$)



1 the digit 1 represents one whole
0.3 the digit 3 represents three tenths
0.07 the digit 7 represents seven hundredths
0.005 the digit 5 represents five thousandths

1.375 = + 0.3 + 0.07 + 0.005

Reading and Writing Decimals

The number of digits after the decimal point tells how to read a decimal number.

0 . __			
one digit	0.4 four tenths	0.5 five tenths	0.7 seven tenths
0 . __ __			
two digits	0.40 forty hundredths	0.05 five hundredths	0.35 thirty-five hundredths
0 . __ __ __			
three digits	0.400 four hundred thousandths	0.005 five thousandths	0.250 two hundred fifty thousandths

---

For decimals greater than one, read the whole number, say "and" for the decimal point, and then read the decimal.

3	.	75
three	*and*	*seventy-five hundredths*
10	.	5
ten	*and*	*five tenths*
200	.	05
two hundred	*and*	*five hundredths*
17	.	345
seventeen	*and*	*three hundred forty-five thousandths*

Say this number: 40.35
Write this number: three hundred five and four tenths

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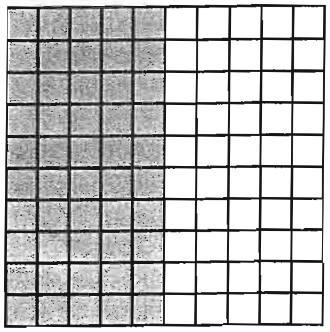
Equivalent Decimals, Fractions, and Percents

(page 1 of 2)

You can describe the shaded part of this 10 × 10 square in different ways.



How many tenths are shaded?
0.5 (5 out of 10 columns are shaded)

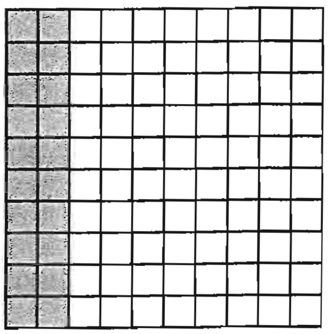
How many hundredths are shaded?
0.50 (50 out of 100 squares are shaded)

These decimals are equal: $0.5 = 0.50$

There are many ways to represent the same part of a whole with decimals, fractions, and percents.

$$0.5 = 0.50 = \frac{1}{2} = \frac{5}{10} = \frac{50}{100} = 50\%$$

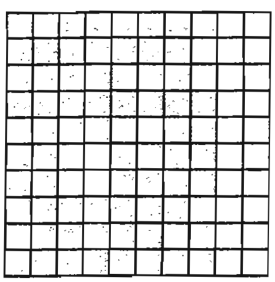
Now look at this 10 × 10 square.



How many tenths are shaded?
0.2 (2 out of 10 columns are shaded)

How many hundredths are shaded?
0.20 (20 out of 100 squares are shaded)

$$0.2 = 0.20 = \frac{2}{10} = \frac{1}{5} = \frac{20}{100} = 20\%$$



How many tenths are shaded?
How many hundredths are shaded?
What fractional parts are shaded?
What percent is shaded?

Equivalent Decimals, Fractions, and Percents

(page 2 of 2)

Find the decimal equivalents for $\frac{1}{8}$, $\frac{4}{8}$, and $\frac{5}{8}$.

Several students used different strategies to find the solution to this problem.

Tavon's solution

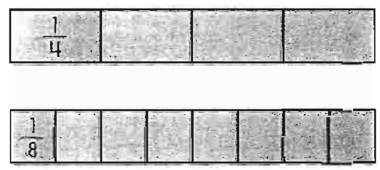
I used my calculator to figure out $\frac{1}{8}$. The fraction $\frac{1}{8}$ is the same as $1 \div 8$, and the answer is 0.125.

$$\frac{1}{8} = \mathbf{0.125}$$

Margaret's solution

I got the same answer a different way.

$\frac{1}{8}$ is half of $\frac{1}{4}$ and $\frac{1}{4} = 25\%$. So, $\frac{1}{8}$ is half of 25%. That's $12\frac{1}{2}\%$, or 0.125.



$$\frac{1}{8} = \mathbf{0.125}$$

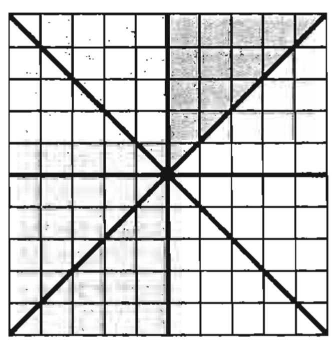
Avery's solution

To solve $\frac{4}{8}$, I just thought about equivalent fractions. $\frac{4}{8}$ is really easy because it is the same as $\frac{1}{2}$.

$$\frac{4}{8} = \frac{1}{2} = \mathbf{0.5}$$

Samantha's solution

I imagined $\frac{5}{8}$ shaded on a 10×10 square. That fills up $\frac{1}{2}$ plus one more eighth.



$$\frac{5}{8} = \frac{1}{2} + \frac{1}{8}$$
$$= 50\% + 12\tfrac{1}{2}\%$$
$$= 62\tfrac{1}{2}\%$$
$$\frac{5}{8} = \mathbf{0.625}$$

Find the decimal equivalents for these fractions:

$\frac{6}{8}$ $\frac{7}{8}$ $\frac{8}{8}$

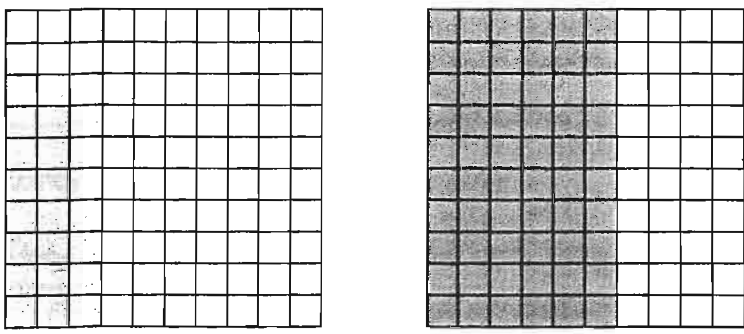
Comparing and Ordering Decimals (page 1 of 2)

Which is larger, 0.35 or 0.6?

Rachel's solution

Rachel used 10 × 10 squares to compare the decimals.

I thought 0.35 was bigger because it has more numbers in it. But when I drew the picture, I saw that 0.6 is the same as $\frac{60}{100}$, which is more than $\frac{35}{100}$.



35 is greater than 6, but 0.35 is not greater than 0.6.

$0.35 = \frac{35}{100}$ $\quad$ $0.6 = \frac{6}{10} = \frac{60}{100}$

0.35 < 0.6

Three students ran a 400-meter race.

Place their times in order from fastest to slowest.

Walter looked at place value to put the times in order.

NAME	TIME (SECONDS)
CHARLES	51.12
MARTIN	50.90
STUART	51.04

The least number of seconds is the fastest time.

Walter's solution

First Place:	Martin, 50.90 seconds	*I looked at the whole number parts. Since 50 < 51, 50.90 is the fastest time.*
Second Place:	Stuart, 51.04 seconds	*Stuart and Charles each finished in a little more than 51 seconds. 4 hundredths is less than 12 hundredths, so Stuart was faster than Charles.*
Third Place:	Charles, 51.12 seconds	

Comparing and Ordering Decimals (page 2 of 2)

What is the order of these decimals from least to greatest?

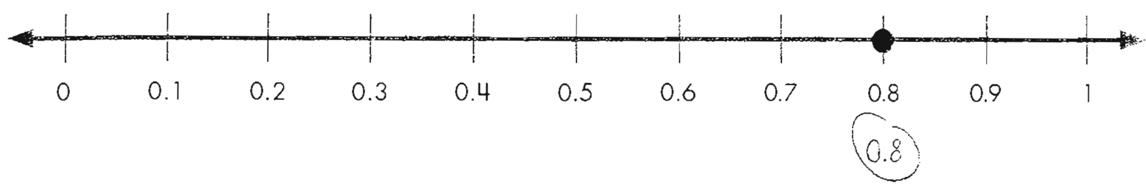
0.8 eight tenths	0.55 fifty-five hundredths	0.625 six hundred twenty-five thousandths

Samantha used a number line to put the decimals in order.

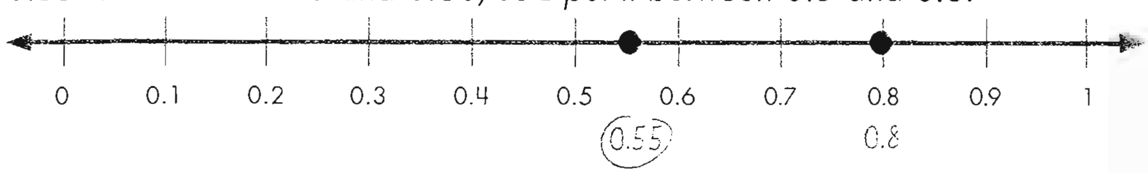
Samantha's solution

I used a number line from 0 to 1.

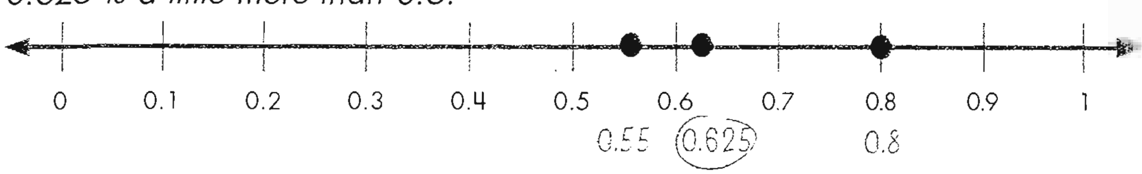
I marked the tenths on the number line and I knew where to put 0.8.



0.55 is between 0.50 and 0.60, so I put it between 0.5 and 0.6.



0.625 is a little more than 0.6.



0.55 < 0.625 < 0.8

Which is larger, 0.65 or 0.4?
Which is larger, 0.4 or 0.375?

Adding Decimals (page 1 of 3)

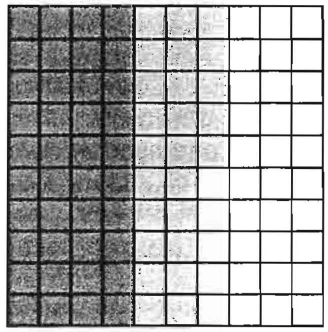
Deon, Alicia, and Zachary used different strategies to add these decimals.

$$0.4 + 0.25 =$$

Deon's solution

I used different colors to shade the decimals on a 10 × 10 square.

*The total is 6 tenths and 5 hundredths, or **0.65**.*



Alicia's solution

0.40	*0.4 is the same as 0.40.*
+ 0.25	
0.65	

0.4 is close to $\frac{1}{2}$ and 0.25 is the same as $\frac{1}{4}$, so I knew the answer should be close to $\frac{3}{4}$, or 0.75.

Zachary's solution

So, I added by place. I added the tenths, and then the hundredths.

0.4 is 4 tenths and 0 hundredths.

0.25 is 2 tenths and 5 hundredths.

0.4 + 0.2 = 0.6

*6 tenths and 5 hundredths is **0.65**.*

Since 25 + 4 = 29, at first I thought the answer would be 0.29, but I could tell from Deon's picture that 0.29 didn't make sense.

Adding Decimals (page 2 of 3)

Shandra, Joshua, Nora, and Lourdes solved this addition problem in different ways.

What is the sum of these decimals?

0.6 six tenths	0.125 one hundred twenty-five thousandths	0.45 forty-five hundredths

Shandra's solution

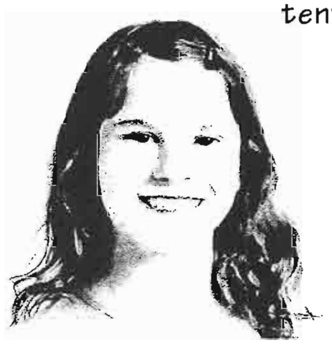
I broke up the numbers and added by place.

First I added all of the tenths.
Next I added the hundredths.
Then I added everything together.

0.6 + 0.1 + 0.4 = 1.1

0.02 + 0.05 = 0.07

1.1 + 0.07 + 0.005 = **1.175**



I knew that the answer would be more than 1 because in the tenths I saw 0.6 and 0.4, which add up to 1.

Joshua's solution

I used equivalents. I just thought of all the numbers as thousandths; then I added them.

0.6 = 0.600

0.45 = 0.450

600 + 450 = 1,050

1,050 + 125 = 1,175

Since 1,000 thousandths is 1, the answer is **1.175.**

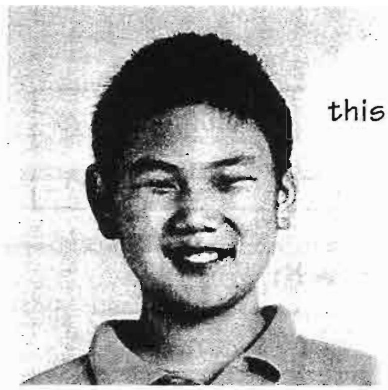
Adding Decimals (page 3 of 3)

0.6 + 0.125 + 0.45 = ?

Walter's solution

I did it kind of like Joshua, but I lined up the numbers and then added.

```
  0.600
  0.125
+ 0.450
  1.100
  0.070
+ 0.005
  1.175
```



You can't just add like this because the decimal place values have to match.

```
    6
  125
+  45
```

Lourdes' solution

I split up 0.45 into 4 tenths and 5 hundredths.

0.45 = 0.4 + 0.05

```
  0.4        0.050
+ 0.6      + 0.125
  1.0        0.175

        1.175
```

You may notice that you are using the same strategies to add decimals that you used to add whole numbers. You can review those addition strategies on pages 8–9 in this handbook.

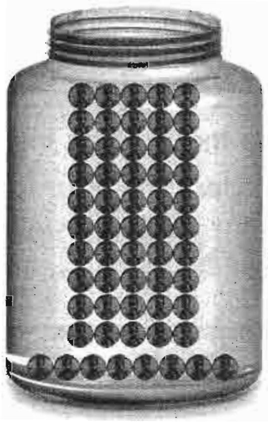
0.65 + 0.3 = ______ **0.375 + 0.2 = ______**

Writing Rules to Describe Change (page 1 of 2)

The rule for the Penny Jar below is:

Start with 8 pennies and add 5 pennies each round.

How many pennies will there be in the jar after 10 rounds?



10	rounds
× 5	pennies per round
50	pennies
+ 8	pennies from the start
58	Total pennies after round 10

These students wrote a rule for the number of pennies for any round using words or an arithmetic expression.

Terrence's rule: You multiply the number of rounds by 5. Then you add 8 because that is the number of pennies in the jar at the beginning.

Janet's rule: Round x 5 + 8

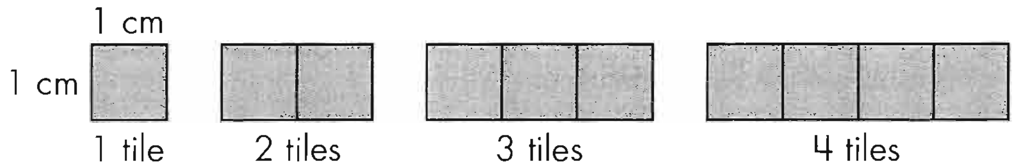
Joshua's rule: $8 + (5 \times n)$ ←

In Joshua's rule, *n* stands for the number of rounds. He could have used a different letter, such as *x* or *r*.

Use one of these rules or your own rule to find out how many pennies will be in the jar after round 30.

Writing Rules to Describe Change (page 2 of 2)

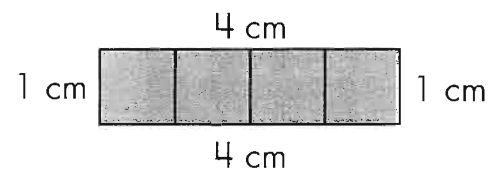
Here is a series of rectangles made out of square tiles.
The sides of each square tile are 1 centimeter.



Some students looked at how the perimeter changed as the rectangle grew.

Number of Square Tiles	Perimeter of Rectangle
1	4
2	6
3	8
4	10
5	12

Perimeter is the measure of the distance around the border of a figure. You can read more about perimeter on page 101.



The perimeter of this rectangle is 10 centimeters.

Some students discussed the rules they wrote for determining the perimeter for any rectangle in this pattern using any number of square tiles.

Stuart: *You double the number of squares and add 2. My rule is* $P = 2n + 2$.

Tamira: *I see it differently. You add 1 to the number of squares and then you double that. My rule is* $P = (1 + n) \times 2$.

Samantha: *My way is almost the same as Stuart's. I add the number of squares to itself, and then add 1 and 1 for the ends. My rule is* $P = n + n + 1 + 1$.

Use one of these rules or your own rule to determine the perimeter of a rectangle in this pattern made of 50 tiles.

SMH

Describing and Summarizing Data

Math Words
- **range**
- **mode**
- **outlier**

Here are some of the observations that Janet and Felix made about their accordion bridge data, given on pages 80 and 81.

Janet noticed the range of this data set.

The data ranged from 15 pennies to 72 pennies. The accordion bridge always held at least 15 pennies. The most pennies it could hold was 72, but that only happened once.

The range is the difference between the highest value and the lowest value in a set of data.

In these data, the range is 57 pennies:

72	−	15	=	57
highest value		lowest value		range

Felix found an interval where most of the data are concentrated.

Ten out of fifteen times, the accordion bridge held between 20 pennies and 39 pennies. That's two thirds of the time.

Janet noticed the mode in this data set.

On the line plot you can see that the bridge held 38 pennies most often, but that only happened 3 out of 15 times.

The mode is the value that occurs most often in a set of data.

Janet noticed an outlier in this data set.

On one trial the bridge held 72 pennies, which is far away from the rest of the data. The rest of the time the bridge held between 15 and 46 pennies.

An outlier is a piece of data that has an unusual value, much lower or much higher than most of the data.

Felix found the median in this data set.

The median is 31 pennies. That means that in half of the trials, the bridge held 31 pennies or more.

The median is the middle value of the data when all the data are put in order.

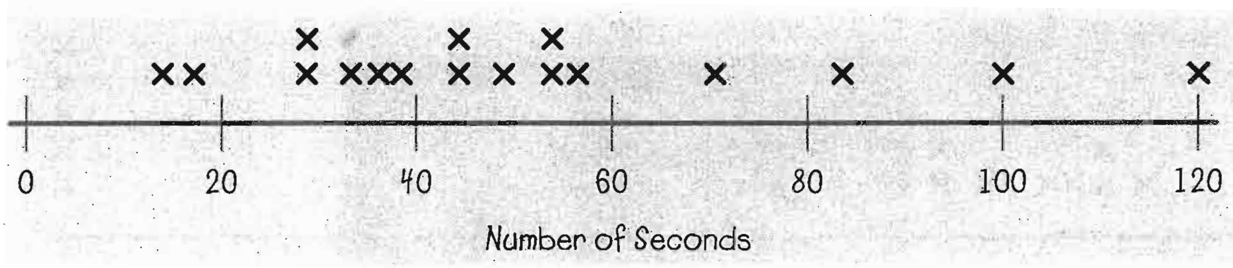
Finding the Median

(page 1 of 2)

Math Words
- **median**

The median is the middle value of the data when all of the data values are put in order.

How long can adults balance on their left feet?



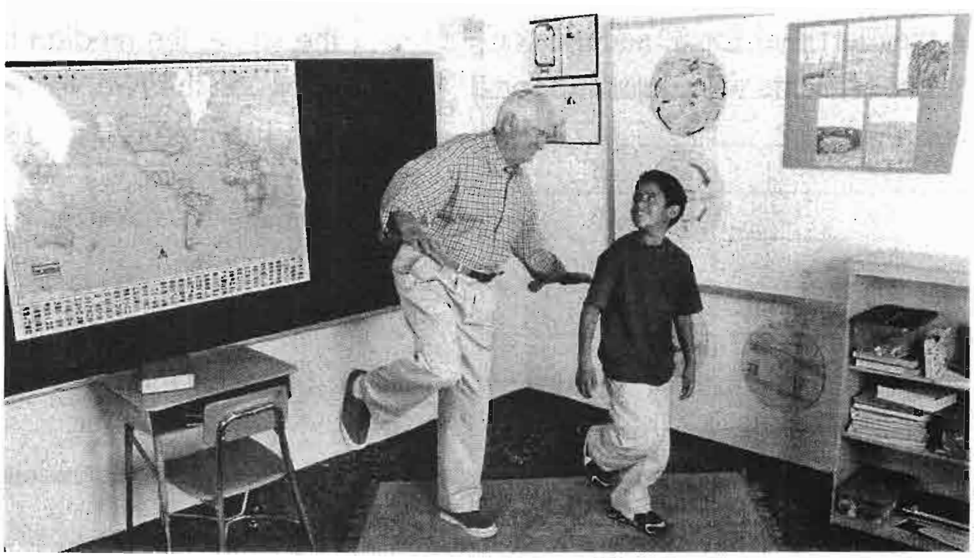
Here are the 17 values listed in order:

15, 17, 29, 29, 31, 35, 38, 45, 45, 49, 53, 53, 55, 70, 82, 100, 120

↑ median

Half of the adults balanced on their left feet for 45 seconds or less, and half of them balanced for 45 seconds or more.

The middle value is 45, so the median value is 45 seconds.

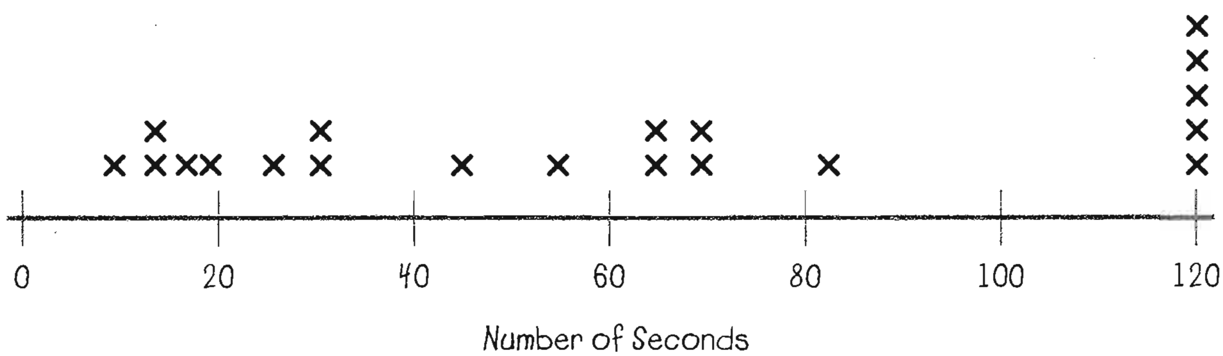


Finding the Median

(page 2 of 2)

When a set of data has an even number of values, the median is between the two middle values.

How long can students balance on their left feet?



Here are the 20 values listed in order:

10, 14, 14, 18, 19, 25, 30, 30, 45, 55, 65, 65, 70, 70, 82, 120, 120, 120, 120, 120

↑ median

There are as many students in the group who balanced on their left feet for 60 seconds or less as there are students who balanced for 60 seconds or more.

Since the middle values are not the same, the median is midway between the two values 55 and 65. The median is 60 seconds.

How would you compare these adults and students? Which group has the better balancers, or are they about the same?

Probability (page 1 of 3)

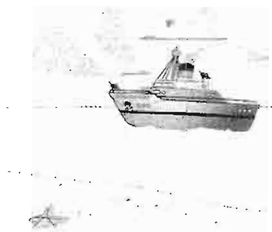
Math Words
- **probability**
- **certain**
- **impossible**

How likely is it . . . ? What are the chances . . . ?

Probability is the study of measuring how likely it is that something will happen. Sometimes we estimate probability based on data and experience about how the world works.

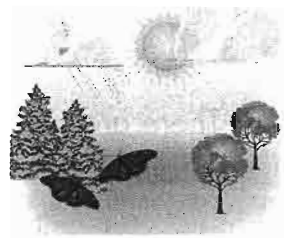
Some future events are impossible, based on what we know about the world.

The entire Pacific Ocean will freeze this winter.



Some future events are certain.

The sun will rise tomorrow.

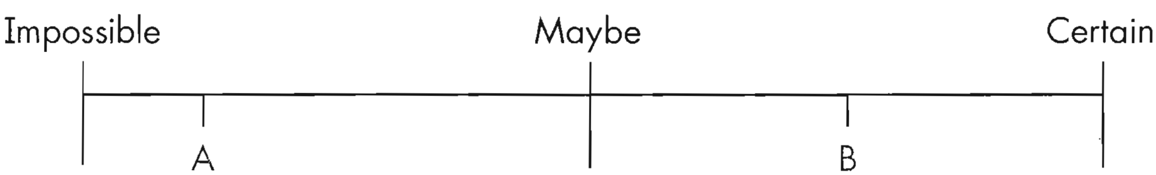


The probability of many other events falls between impossible and certain.

No one in our class will be absent tomorrow.

It will rain next weekend.

Likelihood Line



Describe events that can go at points A and B on the Likelihood Line.

Probability (page 2 of 3)

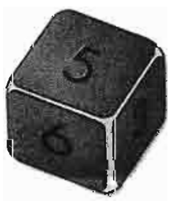
Math Words
- **equally likely**

In some situations, there are a certain number of equally likely outcomes. In these situations, you can find the probability of an event by looking at how many different ways it can turn out.

What will happen if you toss a coin?

There are two possible outcomes. You can get heads or tails. If the coin is fair, there is a 1 out of 2 chance that you will get heads and a 1 out of 2 chance that you will get tails.

What will happen if you roll a number cube marked with the numbers 1, 2, 3, 4, 5, and 6?



There are six possible outcomes. If the number cube is fair, all of the outcomes are equally likely.

The probability of rolling a five is 1 out of 6.

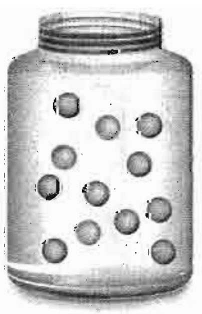
What is the chance of rolling an even number?

1	2
3	4
5	6

There are 3 even numbers out of 6 possibilities. So, there is a 3 out of 6 chance of rolling an even number.

You can also say that this is a 1 out of 2 chance.

What will happen if you pull a marble out of a jar that contains 3 yellow marbles and 9 blue marbles?



There are 12 marbles in the jar. The chance of pulling out a blue marble is 9 out of 12.

You can also say that this is a 3 out of 4 chance.

Probability (page 3 of 3)

In mathematics, you can use numbers from 0 to 1 to describe the probability of an event.

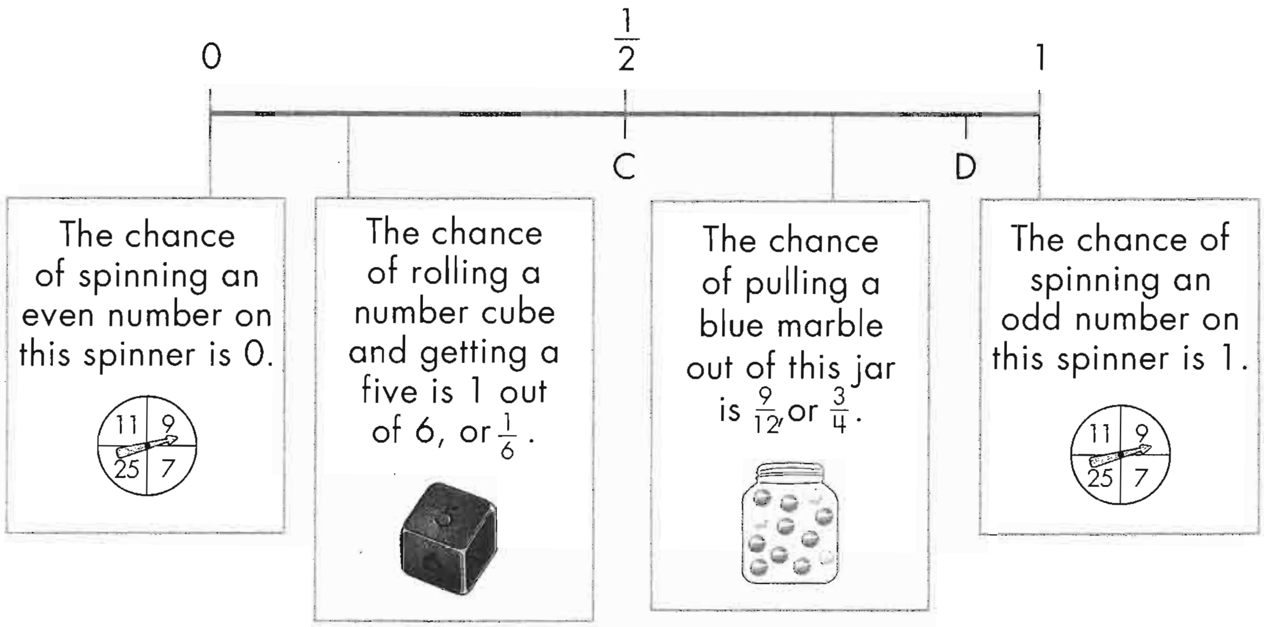
The probability of an impossible event is 0.

The probability of a certain event is 1.

The probability of an event that is equally likely to happen or not happen is $\frac{1}{2}$.

For example, when you flip a fair coin there is a 1 out of 2 chance that you will get heads. The probability of getting heads is $\frac{1}{2}$.

Probabilities can fall anywhere from 0 to 1.



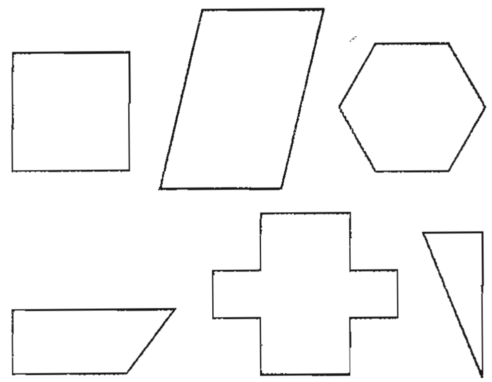
Describe events that can go at points C and D on the line. You can use the idea of a spinner, a number cube, or pulling marbles out of a jar.

Polygons

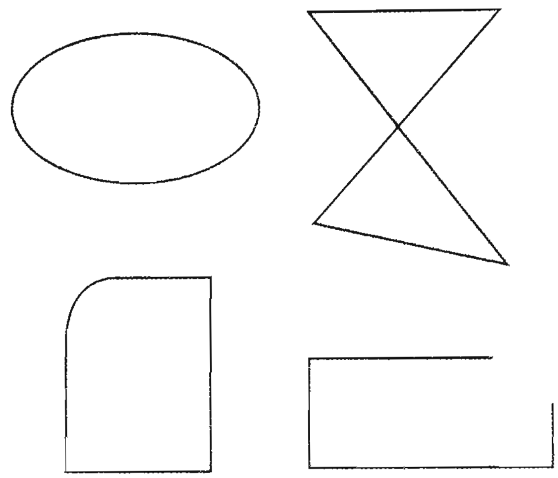
Math Words
- **polygon**
- **two-dimensional (2-D)**

Polygons are closed two-dimensional (2-D) figures with straight sides.

These figures are polygons.



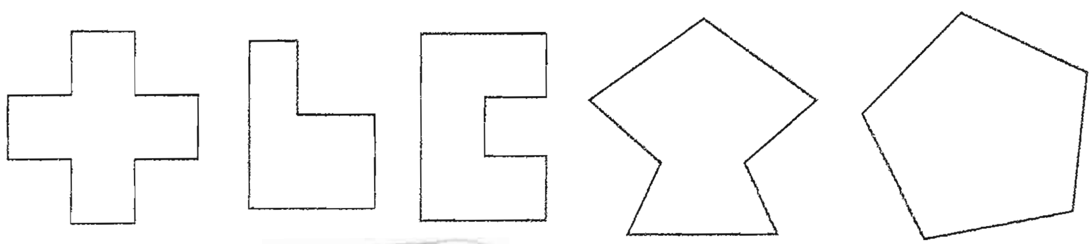
These figures are not polygons.



Polygons are named for the number of sides they have.

3 sides	triangle	8 sides	octagon
4 sides	quadrilateral	9 sides	nonagon
5 sides	pentagon	10 sides	decagon
6 sides	hexagon	11 sides	hendecagon
7 sides	heptagon (or septagon)	12 sides	dodecagon

What is the name of each of these polygons?

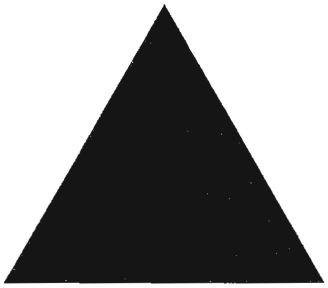


65B

Regular Polygons

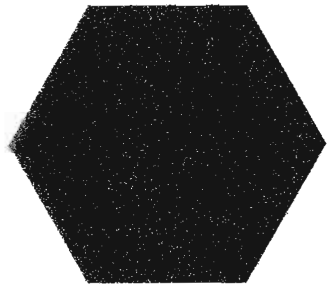
Math Words
- **regular polygon**

Polygons that have equal sides and equal angles are called regular polygons.



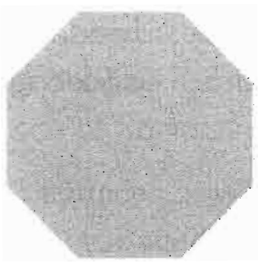
a regular triangle (called an equilateral triangle)

a regular quadrilateral (called a square)

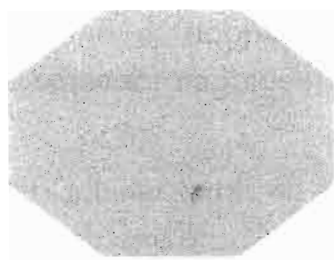


a regular hexagon

Which of these figures are regular polygons? How do you know?



or

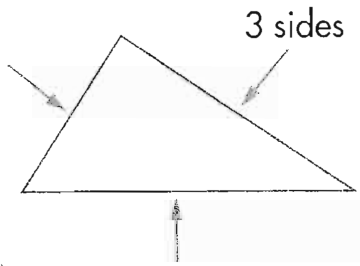


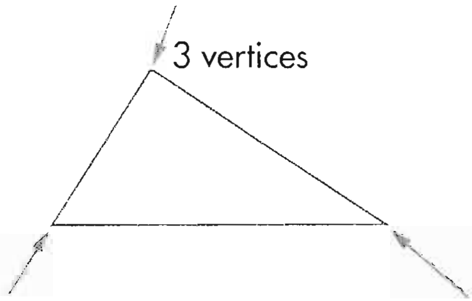
or

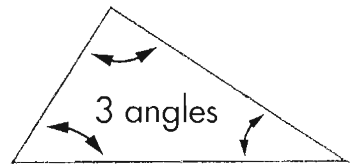
Triangles

Math Words
- **triangle**
- **right triangle**
- **equilateral triangle**

A triangle is a polygon that has:







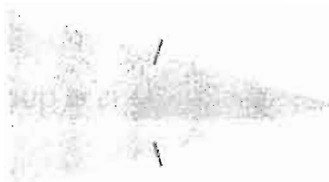
Triangles are described in two ways:

- By the lengths of their sides:

scalene triangle

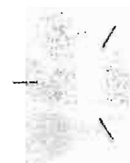
All sides have different lengths.

isosceles triangle



Two sides have the same length.

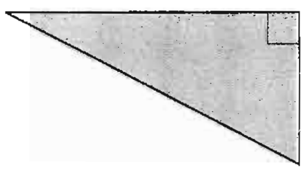
equilateral triangle



All three sides have the same length.

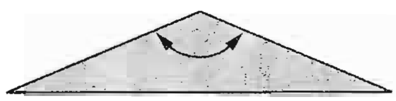
- By the sizes of their angles:

right triangle



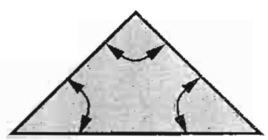
One angle is 90°.

obtuse triangle



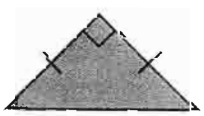
One angle is greater than 90°.

acute triangle



All angles are less than 90°.

This is an isosceles right triangle.



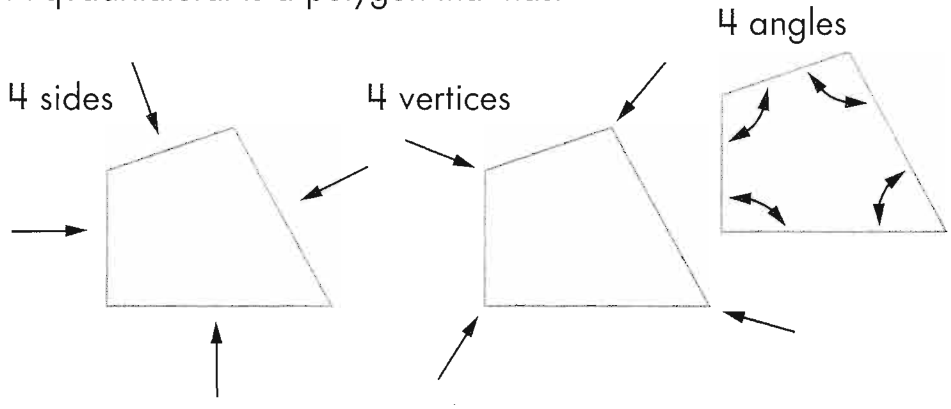
Draw a scalene acute triangle. Draw an obtuse isosceles triangle. Is it possible to draw a right equilateral triangle? Why or why not?

SMH

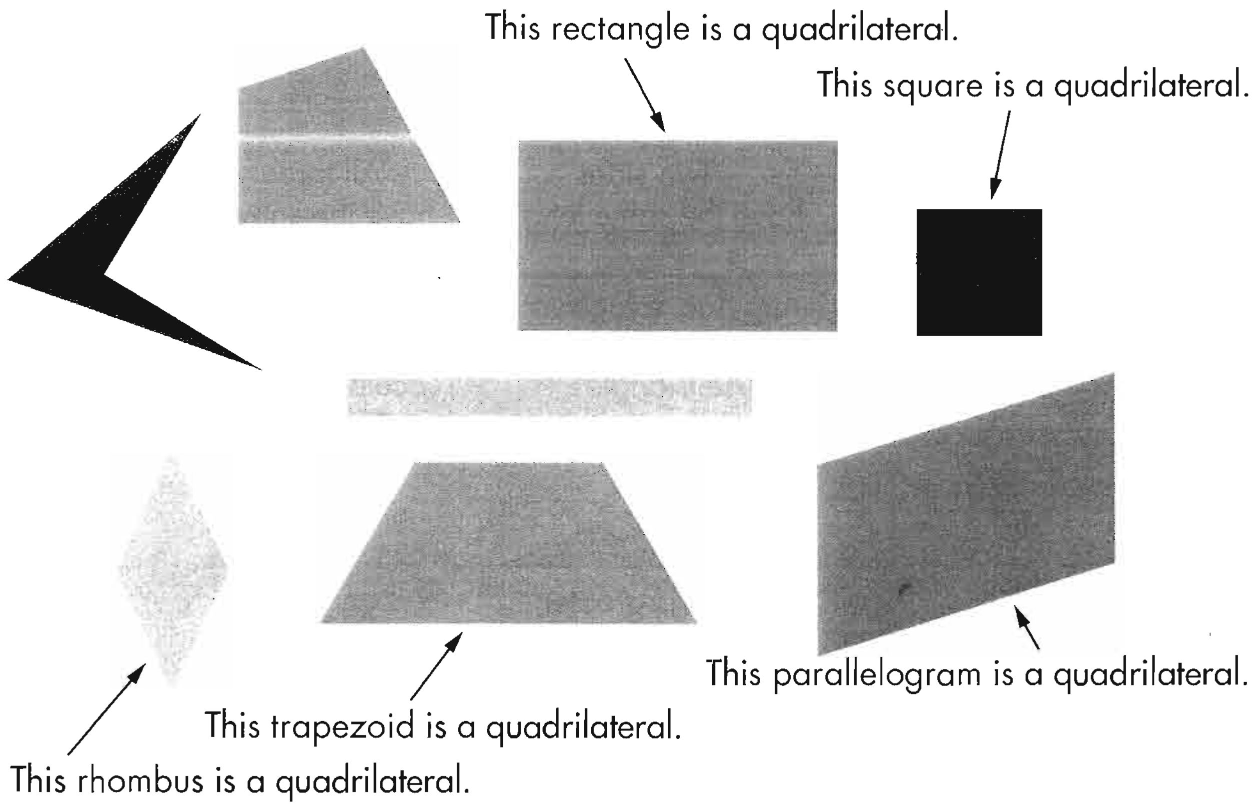
Quadrilaterals (page 1 of 3)

Math Words
- **quadrilateral**

A quadrilateral is a polygon that has:



All of these figures are quadrilaterals. Some quadrilaterals have special names.

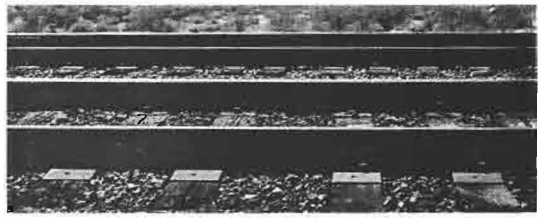


Draw a polygon that is a quadrilateral.
Draw a polygon that is not a quadrilateral.

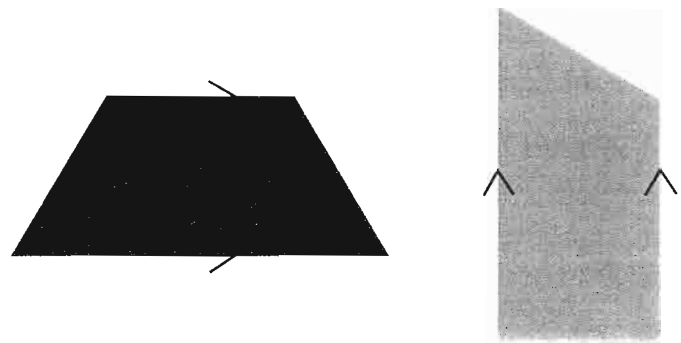
Quadrilaterals (page 2 of 3)

Math Words
- **parallel**
- **trapezoid**
- **parallelogram**

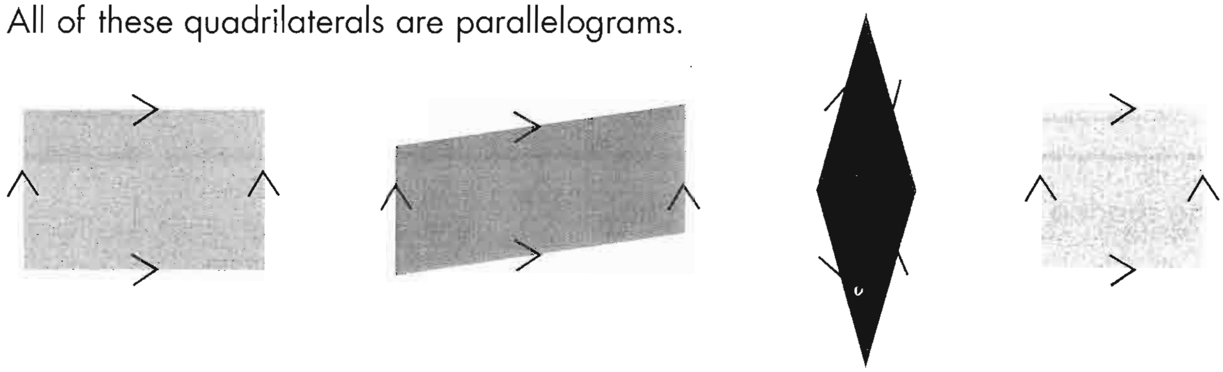
Parallel lines go in the same direction. They run equidistant from one another, like railroad tracks.



Quadrilaterals that have only 1 pair of parallel sides are called trapezoids. Both of these quadrilaterals are trapezoids.



Quadrilaterals that have 2 pairs of parallel sides are called parallelograms. All of these quadrilaterals are parallelograms.



Some quadrilaterals have no parallel sides.

You can use the *LogoPaths* software to draw parallelograms and other polygons.

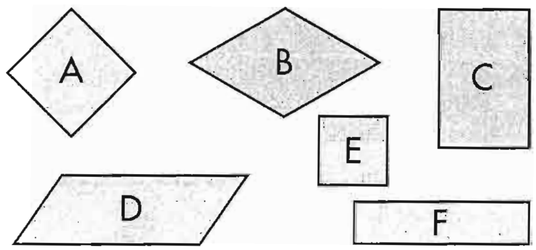
Quadrilaterals (page 3 of 3)

Math Words
- **parallelogram**
- **rectangle**
- **rhombus**
- **square**

Some quadrilaterals can be called many different names.

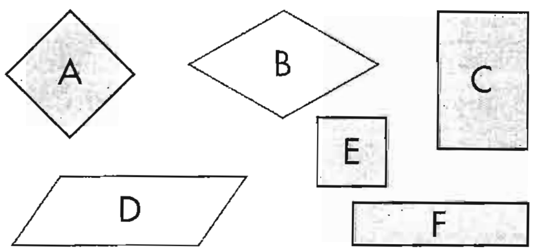
These shaded figures are parallelograms. Each has:

- 2 pairs of parallel sides



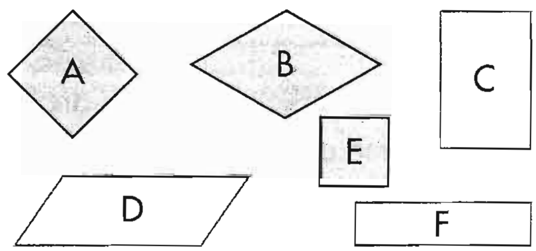
The shaded figures are rectangles. Each has:

- 2 pairs of parallel sides
- 4 right angles



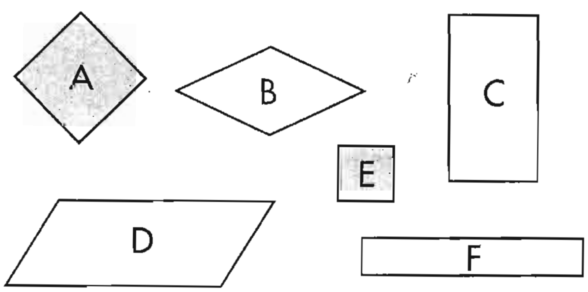
The shaded figures are rhombuses (rhombi). Each has:

- 2 pairs of parallel sides
- 4 equal sides



The shaded figures are squares. Each has:

- 2 pairs of parallel sides
- 4 equal sides
- 4 right angles

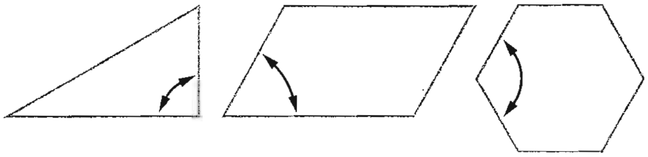


What is the same about rectangles and squares?
What is different about rectangles and squares?

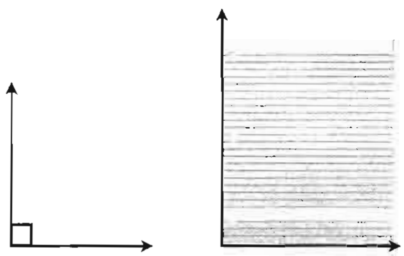
Angles (page 1 of 3)

Math Words
- **angle**
- **degree**
- **right angle**

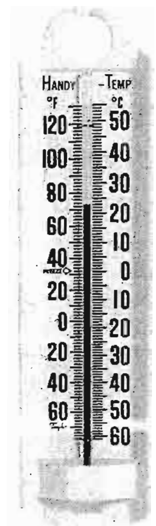
The measure of an angle in a polygon is the amount of turn between two sides.



Angles are measured in degrees. When an angle makes a square corner, like the corner of a piece of paper, it is called a right angle. A right angle measures 90 degrees.

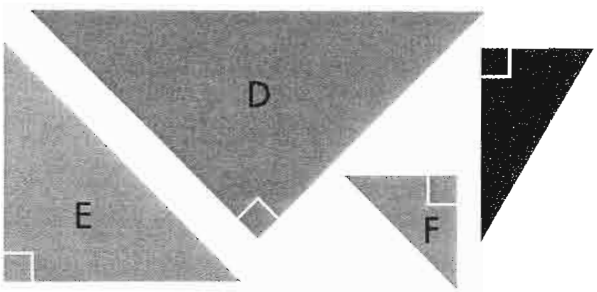


The word *degree* is also a unit that is used to measure temperature.

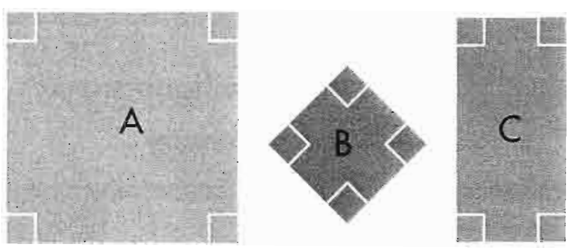


These students are talking about the angles in polygons from their set of Power Polygons™.

Deon: *These triangles all have one 90 degree angle.*



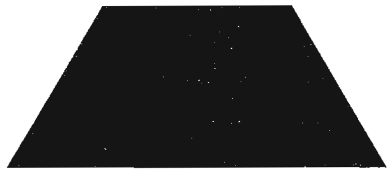
Janet: *All of the angles in all of these rectangles are right angles.*



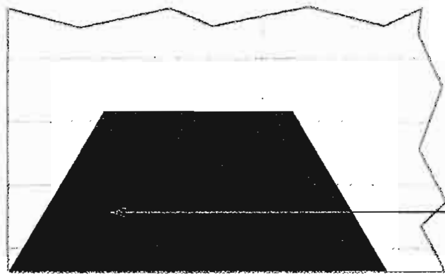
Angles (page 2 of 3)

Math Words
- **acute angle**
- **obtuse angle**

Hana: *None of the angles in this trapezoid is 90 degrees.*

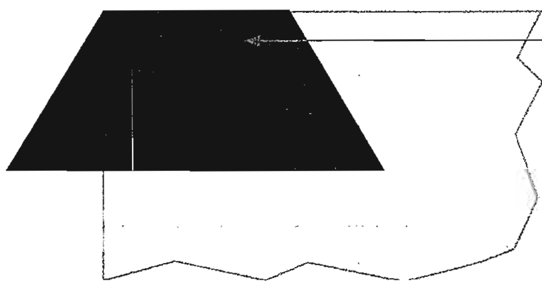


This angle is less than 90 degrees. It is smaller than the corner of the paper.



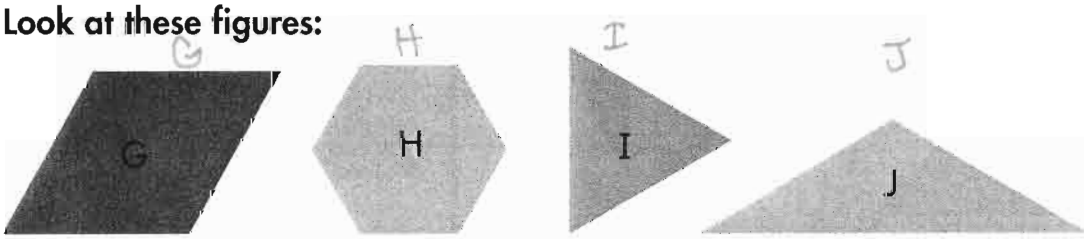
An acute angle is smaller than a right angle.

This angle is greater than 90 degrees. It is larger than the corner of the paper.



An obtuse angle is larger than a right angle.

Look at these figures:

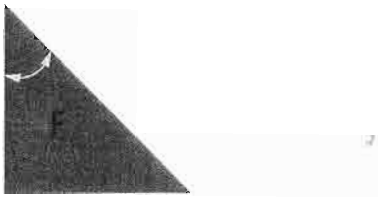


Do you see any 90 degree angles? If so, where?
Do you see any angles less than 90 degrees? If so, where?
Do you see any angles greater than 90 degrees? If so, where?

Angles (page 3 of 3)

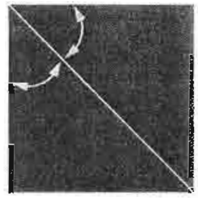
How many degrees are in this angle?

How do you know?



Mitch: *I can use two of these triangles to make a square.*

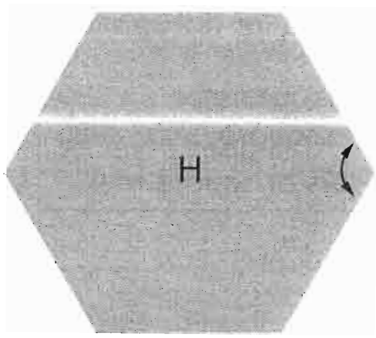
45 + 45 = 90



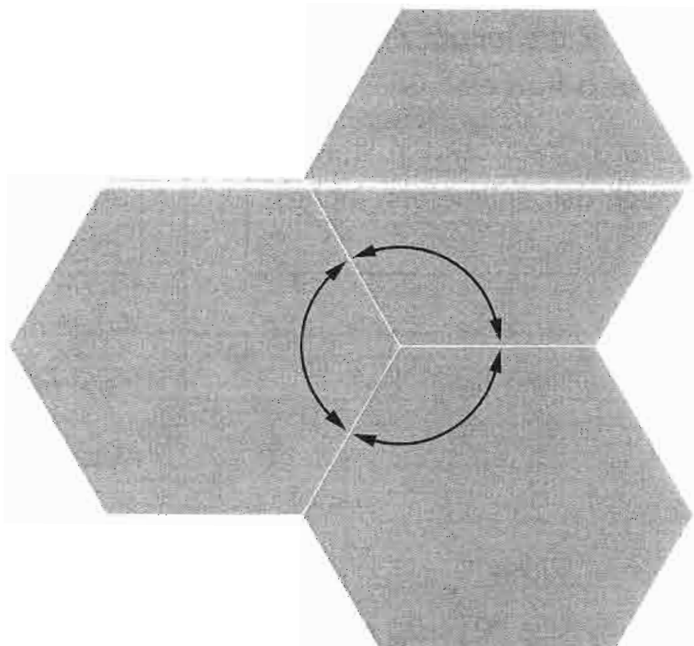
These two angles together make 90°. They are equal, so each angle measures 45°.

How many degrees are in this angle?

How do you know?



Alicia: When I put three of the hexagons together, three of the angles in the middle make a circle.

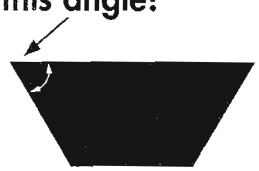


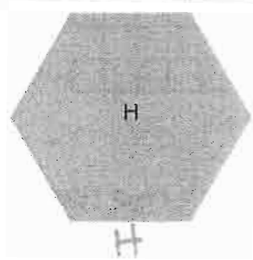
360 ÷ 3 = 120

The circle has 360°, so each angle measures 120°.

You can use the *LogoPaths* software to solve problems about angles.

How many degrees are in this angle?
How do you know?

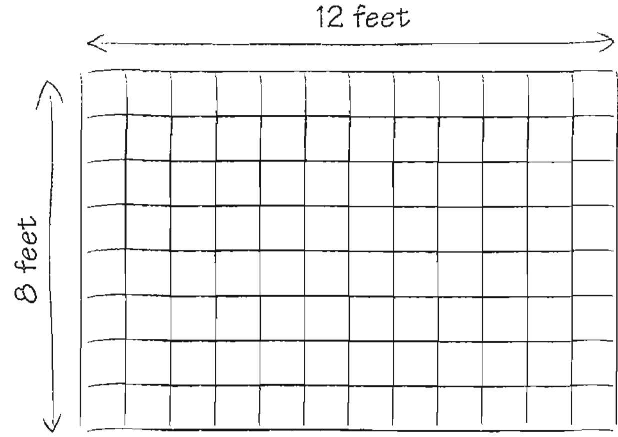




Perimeter and Area

Math Words
- **perimeter**
- **area**

Lourdes and her father are building a patio. The patio is made up of 1-foot square tiles. They are also building a fence around the patio. Here is a sketch of their patio design.



Lourdes and her father need to use two different measurements for their patio project.

Perimeter is the length of the border of a figure. Perimeter is measured in linear units such as centimeters, inches, or feet.

What is the perimeter of the patio?

How long will the fence be?

Cecilia:

8 + 12 + 8 + 12 = 40

perimeter = 40 feet

The fence will be **40 feet** long

Area is the measure of a 2-D surface, for example the amount of flat space a figure covers. Area is measured in square units, such as square centimeters or square feet.

What is the area of the patio?

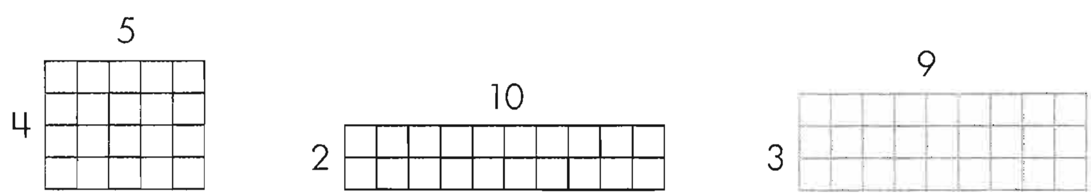
How many square tiles do they need?

Mitch:

8 x 12 = 96

Area = 96 square feet

They need **96 tiles**.



Which two rectangles have the same area?
Which two rectangles have the same perimeter?

Volume of Rectangular Prisms (page 1 of 2)

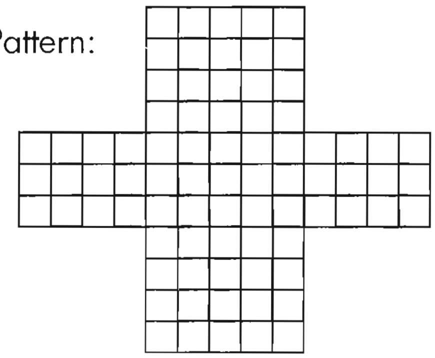
Math Words
- **volume**

Volume is the amount of space a 3-D object occupies. You can think of the volume of a box as the number of cubes that will completely fill it.

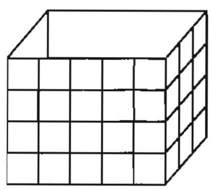
Both Olivia and Joshua solved this problem about the volume of a box.

How many cubes will fit in this box?

Pattern:



Picture:

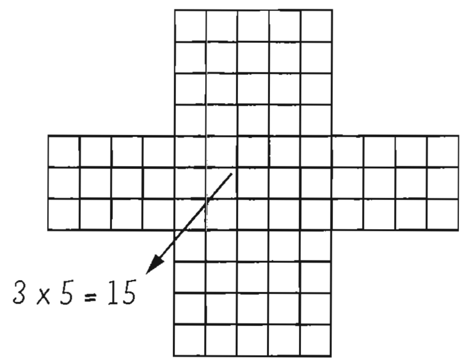


Olivia: There will be 15 cubes on the bottom layer of the box.

When you fold up the sides of the pattern, there will be four layers.

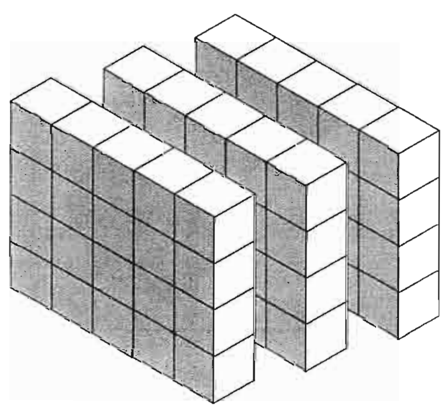
4 x 15 = 60

The box will hold **60 cubes**.



Joshua: The front of the box is 4 by 5, so there are 20 cubes in the front of the box.

The box goes back 3 slices, so 20, 40, **60 cubes** will fit in the box.



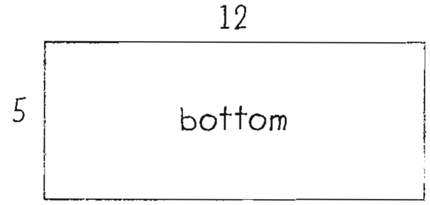
Volume of Rectangular Prisms (page 2 of 2)

Martin solved this problem:

The bottom of a box is 12 units by 5 units.
The box is 8 units high. What is the volume of the box?

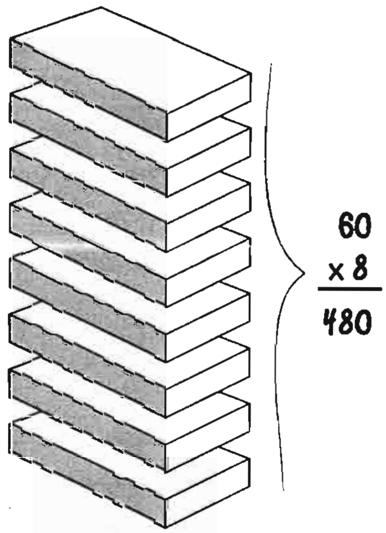
Martin's solution

The bottom layer of the box will have 60 cubes because 12 x 5 = 60.



Since the box has 8 layers, the total number of cubes is 60 x 8.

So, the volume of the box is **480 cubes**.

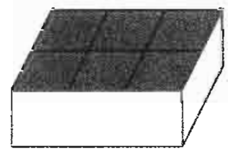


Write a strategy for finding the volume of a rectangular prism. Think about how you can determine the number of cubes that fit in a box, whether you start with the box pattern, the picture of a box, or a written description of the box.

Changing the Dimensions and Changing the Volume

Company A and Company B both make identical boxes that have a volume of 6 cubes.

Original Box Design

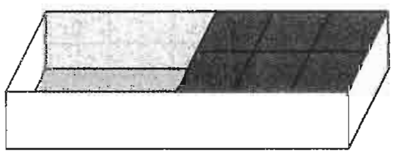


Dimensions: 3 × 2 × 1, holds 6 cubes

Each company has a plan to change the design of the box.

Company A plans to make a box that will hold twice as many cubes.

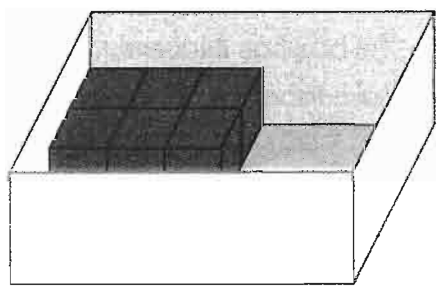
New Box Design: Company A



Dimensions: 6 × 2 × 1, holds 12 cubes

Company B plans to make a box with double the dimensions.

New Box Design: Company B



Dimensions: 6 × 4 × 2, holds 48 cubes

Four students discussed how the volume of each new box compares to the volume of the original box.

Company A

Alicia: *The volume of Company A's new box is twice the volume of the original box.*

Olivia: *Only one dimension changed. The 3 doubled to be a 6.*

Company B

Stuart: *Company B's new box will hold 8 times as many cubes as the original box.*

Tavon: *All three of the dimensions were multiplied by 2.*

Design a different box for Company A that will also hold twice as many cubes as the original 3 × 2 × 1 box.

SMH

Standard Cubic Units

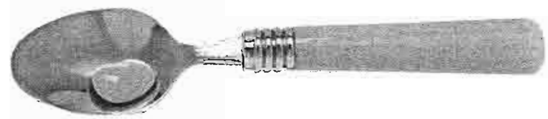
(page 1 of 2)

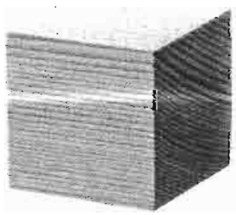
Math Words
- **cubic centimeter**
- **cubic inch**
- **cubic foot**
- **cubic meter**
- **cubic yard**

Volume is measured in cubic units.

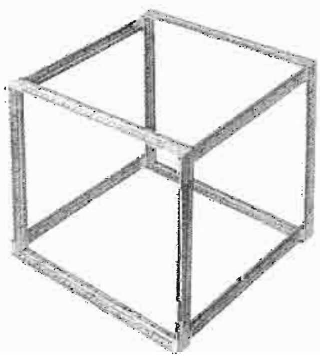
cubic centimeter	length of an edge	area of a face	volume of the cube
	1 centimeter (1 cm)	1 square centimeter ($1\ cm^2$)	1 cubic centimeter ($1\ cm^3$)

A cubic centimeter is about the size of a bean.



cubic inch	length of an edge	area of a face	volume of the cube
	1 inch (1 in.)	1 square inch ($1\ in.^2$)	1 cubic inch ($1\ in.^3$)

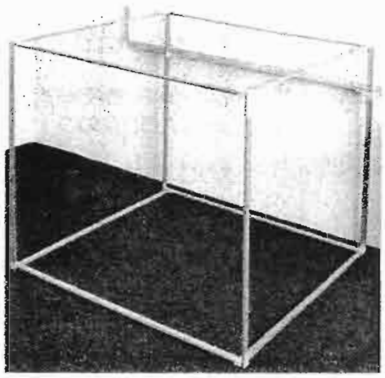
A cubic inch is about the size of a marshmallow.

cubic foot	length of an edge	area of a face	volume of the cube
	1 foot (1 ft)	1 square foot ($1\ ft^2$)	1 cubic foot ($1\ ft^3$)

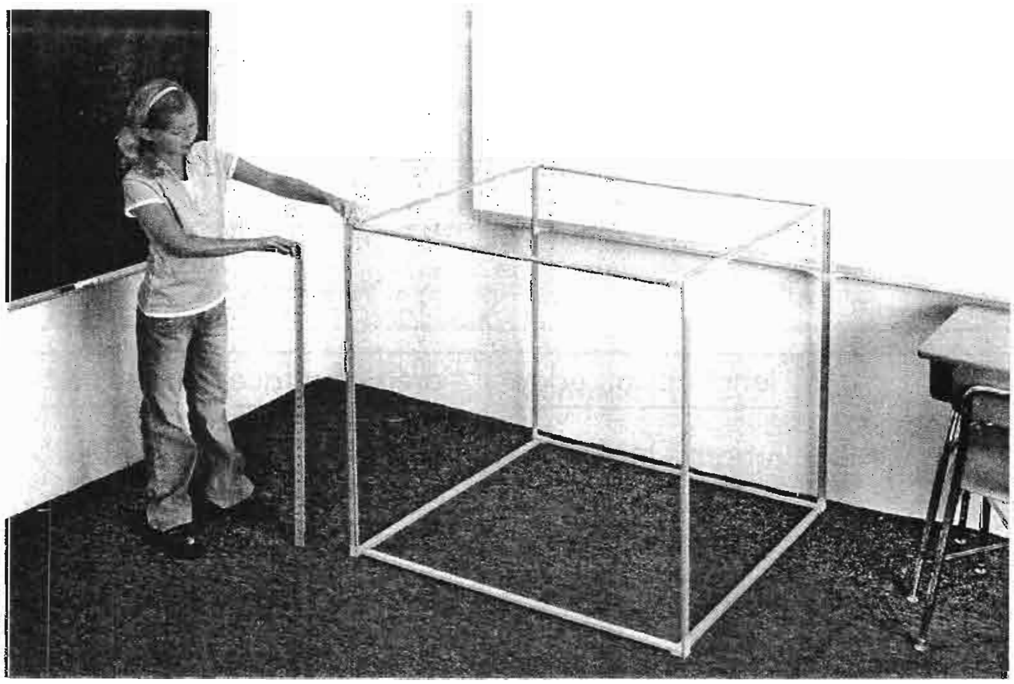
A cubic foot is about the size of a boxed basketball.

Standard Cubic Units

(page 2 of 2)

cubic meter	length of / an edge	area of / a face	volume of / the cube
	1 meter (1 m)	1 square meter (1 m^2)	1 cubic meter (1 m^3)

Since a yard is a little shorter than a meter,
a cubic yard is a little smaller than a cubic meter.



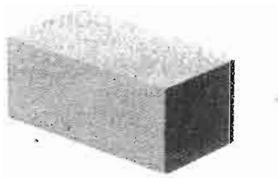
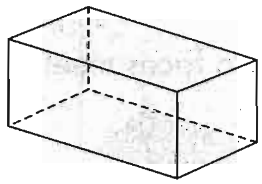
Which unit of measure would you use to find the volume of:
A bathtub? Your kitchen? A brick?

Geometric Solids (page 1 of 4)

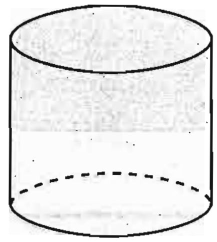
Math Words
- **geometric solid**

Here are some examples of geometric solids. These figures have three dimensions: length, width, and height.

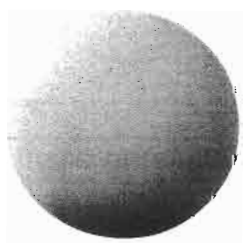
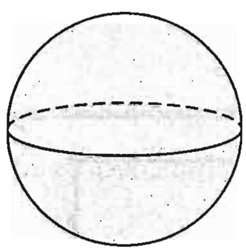
rectangular prism

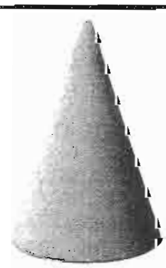
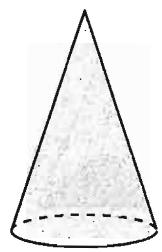

cylinder

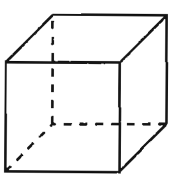
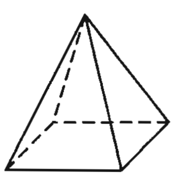
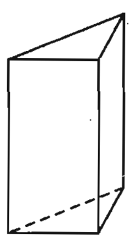

sphere


cone


What real world objects are shaped like these geometric solids?

Geometric Solids (page 2 of 4)

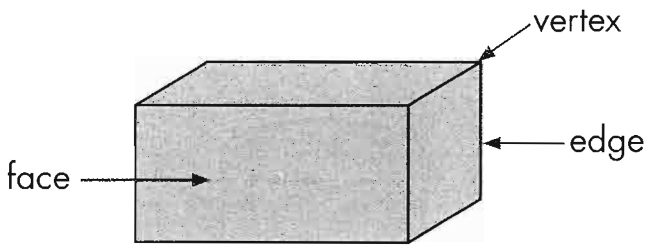
Math Words
- **face**
- **edge**
- **vertex**
- **vertices**

One way to describe a geometric solid is to identify the number of faces, edges, and vertices.

A face is a 2-D figure that makes up a flat surface of a 3-D solid.

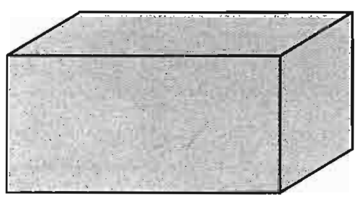
An edge is a line segment where two faces meet.

A vertex is the point at a corner where edges meet.



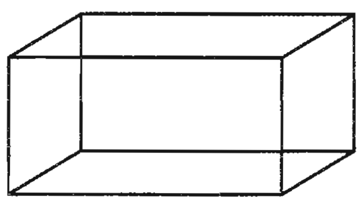
A rectangular prism has:

6 faces

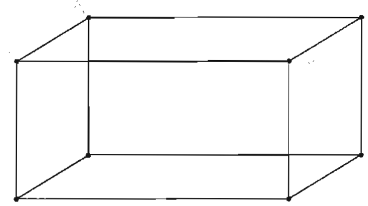


(You cannot see all of the faces in this picture.)

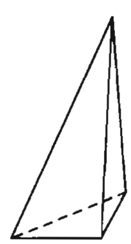
12 edges



8 vertices



How many faces does this triangular pyramid have?
What do the faces look like?
How many edges does it have?
How many vertices does it have?

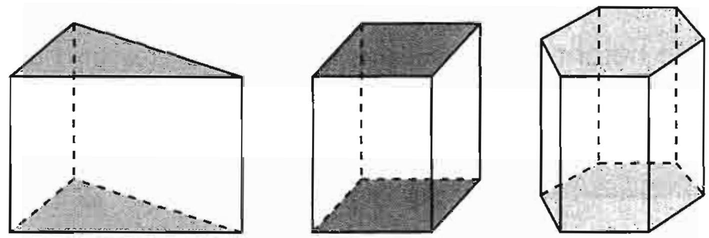


Geometric Solids (page 3 of 4)

Math Words
- **base**

All of these geometric solids are called prisms.

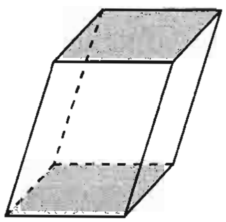
The "top" and "bottom" faces of a prism are called bases.



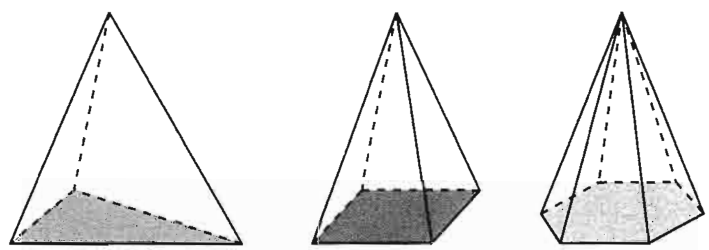
The bases of each prism match one another.

The faces on the sides of these prisms are all rectangles.

Some prisms, like this one, have faces that are parallelograms that are not rectangles.



All of these geometric solids are called pyramids.



The base of each pyramid is a polygon.

There is a point at the top of each pyramid.

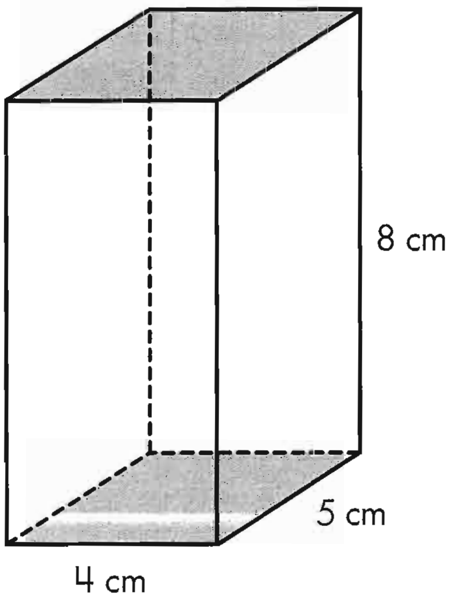
The faces on the sides of the pyramids are all triangles.

Describe the difference between a prism and a pyramid.

Geometric Solids (page 4 of 4)

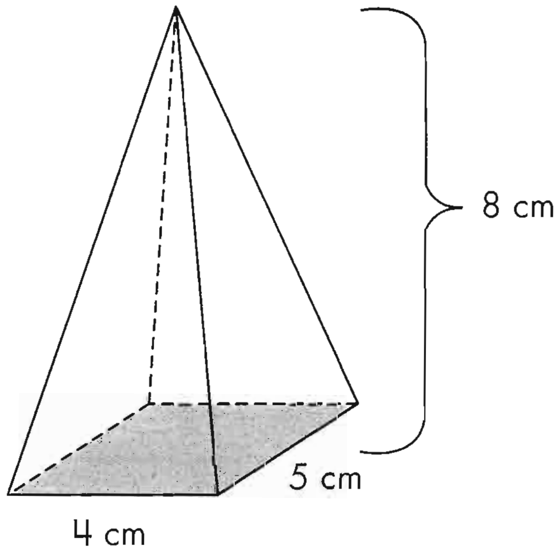
The base of this rectangular prism measures 4 centimeters by 5 centimeters.

The height measures 8 centimeters.

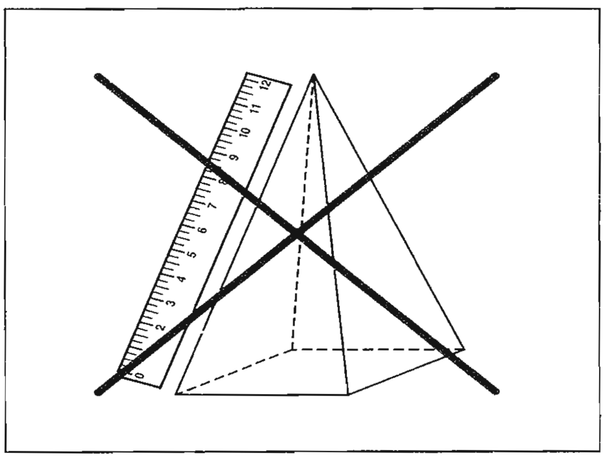


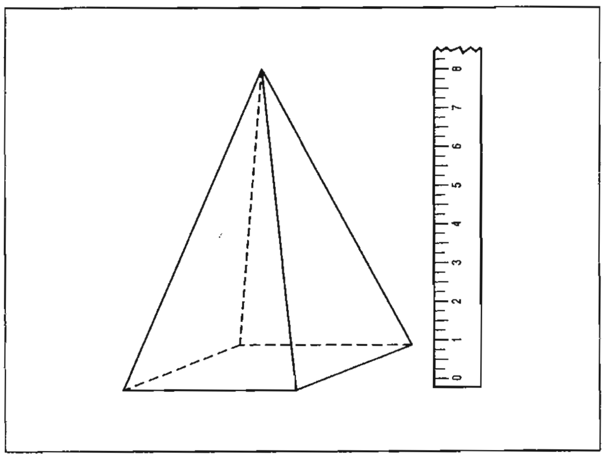
The base of this rectangular pyramid measures 4 centimeters by 5 centimeters.

The height measures 8 centimeters.



Note: The height of the pyramid is measured vertically from the base, not along the slope of the side.





What is the volume of the rectangular prism above?
How do you think the volume of this rectangular pyramid compares to the volume of the rectangular prism above? How could you find out?